International Review of Fuzzy Mathematics, Volume 13, No. 2, (December 2018)

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A MULTI-ATTRIBUTE DECISION MAKING PROBLEM IN A SINGLE VALUED NEUTROSOPHIC METHOD

Abstract: *In this paper we introduce an approach for solving multi attribute decision making problems in which there are several decision makers who independently bring forth their choices. Also we have defined score and accuracy function for hexagonal neutrosophic sets and a single valued neutrosophic fuzzy weighted averaging operator and weighted geometric averaging operators. An example has been provided to validate the proposed approach for multi attribute decision making problem.*

Keywords: *Accuracy function, Score function, Single valued neutrosophic sets, Multi attribute decision making.*

1. INTRODUCTION

Neutrosophic fuzzy decision making is an important part of decision making under uncertainty, which is based on preference order. Smarandache [6] who said 'The goal is to enlargement of the artistic sphere through non-artistic elements. But especially the counter-time, counter- sense creation. Also to experiment", originally gave a concept of neutrosophic probability, set and logic, which is a part of generalized fuzzy sets. Based on fuzzy sets Zadeh [17], introduced interval valued fuzzy sets. Turksen[8] introduced intuitionstic fuzzy sets, and later the interval valued fuzzy sets was introduced by Attnassov[1] and Gargov[2]. As a next step neutrosophic fuzzy numbers Wang *et al.*; [9] presented single valued neutrosophic set and interval valued neutrosophic sets which are subclasses of neutrosophic sets presented by Smarandache [7] .The neutrosophic sets has the components T,I,F which represents the True, Indeterminacy and false values. Biswas *et al.*; [3] established a single valued neutrosophic method with unknown weight information. Biswas *et al.*; [4] also proposed triangular fuzzy neutrosophic set solving MADM problems using aggregate operators. Jun Ye [5] proposed trapezoidal neutrosophic numbers and its application in multi attribute decision making problems. This paper is organized as follows. The definition of intuitionstic fuzzy sets, neutrosophic sets, single valued neutrosophic sets, Interval valued neutrosophic sets, Hexagonal fuzzy neutrosophic sets and some basic operators on them are given. An application problem is given to analyze the proposed method at the end.

2. PRELIMINARIES

2.1. Intuitionistic Fuzzy set

An Intuitionistic Fuzzy Set (IFS) $\tilde{A}^I$ in X is defined as an object of the form $\tilde{A}^I = \{\langle x, \mu_{\tilde{A}^I}(x), \nu_{\tilde{A}^I}(x) \rangle : x \in X\}$ where the functions $\mu_{\tilde{A}^I} : X \to [0,1]$ and $\nu_{\tilde{A}^I} : X \to [0,1]$ define the degree of the membership and the degree of non membership of the element $x \in X$ in $\tilde{A}^I$, $0 \le \mu_{\tilde{A}^I}(x) + \nu_{\tilde{A}^I}(x) \le 1$.

2.2. Intuitionistic Fuzzy number

An Intuitionistic Fuzzy Number (IFN) $\tilde{A}^I$ is

i) an intuitionistic fuzzy subset of the real line,

ii) convex for the membership function $\mu_{\tilde{A}^I}(x)$,

(i.e.) $\mu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \ge \min(\mu_{\tilde{A}^I}(x_1), \mu_{\tilde{A}^I}(x_2))$, for every $x_1, x_2 \in R$, $\lambda \in [0,1]$.

iii) concave for the membership function $\nu_{\tilde{A}^I}(x)$, that is, $\nu_{\tilde{A}^I}(\lambda x_1 + (1-\lambda)x_2) \le \max(\nu_{\tilde{A}^I}(x_1), \nu_{\tilde{A}^I}(x_2))$ for every $x_1, x_2 \in R$, $\lambda \in [0,1]$.

iv) normal, that is, there is some $x_0 \in R$ such that $\mu_{\tilde{A}^I}(x_0) = 1$, $\nu_{\tilde{A}^I}(x_0) = 0$.

Definition 2.3 Hexagonal Intuitionistic Fuzzy Set

Let X be a universe of discourse. A hexagonal intuitionstic fuzzy set $\tilde{A}$ in X is defined as $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x) / x \in X / \rangle\}$ where, $\mu_{\tilde{A}}(x) \subset [0,1]$ *and*, $\nu_{\tilde{A}}(x) \subset [0,1]$ are two hexagonal fuzzy numbers with

$\mu_{\tilde{A}}(x) = \left(\mu^1_{\tilde{A}}(x), \mu^2_{\tilde{A}}(x), \mu^3_{\tilde{A}}(x), \mu^4_{\tilde{A}}(x), \mu^5_{\tilde{A}}(x), \mu^6_{\tilde{A}}(x)\right): X \to [0,1]$ and

$\nu_{\tilde{A}}(x) = \left(\nu^1_{\tilde{A}}(x), \nu^2_{\tilde{A}}(x), \nu^3_{\tilde{A}}(x), \nu^4_{\tilde{A}}(x), \nu^5_{\tilde{A}}(x), \nu^6_{\tilde{A}}(x)\right): X \to [0,1]$ with the condition that $0 \le \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \le 1$

Definition 2.4 Operation on hexagonal Intuitionistic fuzzy numbers

$$Let\ \ \tilde{A}_1 = \left[(a_1, b_{1,}, c_{1,}, d_{1,}, e_{1,}, f_{1,}),\ (l_{1,}, m_{1,}, n_{1,}, p_{1,}, q_1, r_{1,})\right]\ and$$
$$\tilde{A}_2 = [(a_2, b_2, c_2, d_2, e_2, f_2),\ (l_2, m_2, n_2, p_2, q_2, r_2)]$$

be two hexagonal intuitionstic fuzzy numbers then the following operational rules holds.

$$1. \tilde{A}_1 \oplus \tilde{A}_2 = \left\langle \begin{matrix} (a_1 + a_2 - a_1 a_2, b_1 + b_2 - b_1 b_2, c_1 + c_2 - c_1 c_2, d_1 + d_2 - d_1 d_2, e_1 + e_2 - e_1 e_2, f_1 + f_2 - f_1 f_2), \\ (l_1 l_2, m_1 m_2, n_1 n_2, p_1 p_2, q_1 q_2, r_1 r_2) \end{matrix} \right\rangle$$

$$2.\ \tilde{A}_1 \otimes \tilde{A}_2 = \left\langle \begin{matrix} (a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2, e_1 e_2, f_1 f_2), \\ (l_1 + l_2 - l_1 l_2, m_1 + m_2 - m_1 m_2, n_1 + n_2 - n_1 n_2, p_1 + p_2 - p_1 p_2, q_1 + q_2 - q_1 q_2, r_1 + r_2 - r_1 r_2) \end{matrix} \right\rangle$$

$$3.. \lambda \tilde{A}_1 = \left\langle (1-(1-a_1)^\lambda, 1-(1-b_1)^\lambda, 1-(1-c_1)^\lambda, 1-(1-d_1)^\lambda, 1-(1-e_1)^\lambda, 1-(1-f_1)^\lambda), (l_1^\lambda, m_1^\lambda, n_1^\lambda, p_1^\lambda, q_1^\lambda, r_1^\lambda) \right\rangle$$

$$4.\ (\tilde{A}_1)^\lambda = \left\langle \begin{matrix} (a_1^\lambda, b_1^\lambda, c_1^\lambda, d_1^\lambda, e_1^\lambda, f_1^\lambda), \\ (1-(1-l_1)^\lambda, 1-(1-m_1)^\lambda, 1-(1-n_1)^\lambda, 1-(1-p_1)^\lambda, 1-(1-q_1)^\lambda, 1-(1-r_1)^\lambda), \lambda \ge 0 \end{matrix} \right\rangle$$

Definition 2.5 Let $\tilde{A} = \langle (a,b,c,d,e,f),(l,m,n,p,q,r) \rangle$ be a hexagonal intuitionstic fuzzy number. Then the score function of hexagonal intuitionstic fuzzy number is defined by

$$S(\tilde{A}) = \frac{a+b+c+d+e+f}{6} - \frac{l+m+n+p+q+r}{6}, S(\tilde{A}) \in [0,1]$$

Definition 2.6: Let $\tilde{A} = \langle (a,b,c,d,e,f),(l,m,n,p,q,r) \rangle$ be a hexagonal intuitionstic fuzzy number. Then the accuracy function of hexagonal intuitionstic number is defined by

$$H(\tilde{A}) = \frac{a+b+c+d+e+f}{6} + \frac{l+m+n+p+q+r}{6}, H(\tilde{A}) \in [0,1]$$

3. NEUTROSOPHIC SETS

Definition 2.7 : [7] Let X be a space of points (objects),with a generic element in X denoted by x and $x \in X$. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $l_A(x)$ and a falsity-membership function $F_A(x)$ then $T_A(x)$, $l_A(x)$ and $F_A(x)$ are real standard or non-standarad subsets of $]0^-,1^+[$. That is $T_A(x)$: $]0^-,1^+[$, $l_A(x) : X \to]0^-,1^+[$ and $F_A(x) : X \to]0^-,1^+[$.

There is no restriction on the sum of $T_A(x)$, $l_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup l_A(x) + \sup F_A(x) \leq 3^+$

Definition 2.8: [7]. The complement of neutrosophic set A is denoted by A^c and is defined as $T_A^C(x) = \{1^+\} \ \theta \ T_A(x),\ l_A^C(x) = \{1^+\} \ \theta \ l_A^C(x)$ and $F_A^C(x) = \{1^+\} \theta \ F_A(x) \ \ for \ \ all \ \ x \in X$.

3.1. Single valued neutrosphic sets

A single valued neutrosophic set has been defined in [25] as follows:

Definition: 2.9 [9] Let X be a universe of discourse. A single valued neutrosphic set A over X is an object having the form $A = \{\langle x, u_A(x), \omega_A(x), v_A(x) \rangle : x \in X\}$ Where

$\mu_A(x) : X \to [0,1],\ \omega_A(x) : X \to [0,1]$ and $v_A(x) : X \to [0,1]$, with $0 \leq \mu_A(x) + \omega_A(x) + v_A(x) \leq 3$ for $x \in X$.

The intervals $\mu_A(x), \omega_A(x)$ *and* $v_A(x)$ denote the truth-membership degree, the indeterminacy-membership degree and the falsity membership degree of x to A, respectively.

Definition 2.10: [9] The complement of an SVNS A is denoted by A^C and is defined as $\mu_A^C(x) = v(x), \quad \omega_A^C(x) = 1 - \omega_A(x), v_A^C(x) = \mu(x),$ for all $x \in X$. That is, $A^C = \{\langle x, v_A(x), 1 - \omega_A(x), \mu_A(x) \rangle : x \in X\}$.

Definition 2.11: [9] A single valued neutrosophic set A is contained in the other SVNS B, $A \subseteq B$ iff, $(i)\, \mu_A(x) \le \mu_B(x)$ $(ii)\, \omega_A(x) \ge \omega_B(x)$ $(iii)\, v_A(x) \ge v_B(x)$ for all $x \in X$.

Definition 2.12: Let $A_K(k = 1,2,.....n) \in SVNS(X)$. The single valued neutrosophic weighted average operator is defined by

$$F_\omega = (A_1, A_2, ...A_n) = \sum_{k=1}^{n} \omega_k A_k =$$

$$\left(1 - \prod_{k=1}^{n} (1 - \mu_{A_k}(x))^{\omega_k}, \prod_{k=1}^{n} (w_{A_k}(x))^{\omega_k}, \prod_{k=1}^{n} (v_{A_k}(x))^{\omega_k} \right)$$

Where ω_k is the weight of $A_k(k = 1,2,....n)$, $\omega_k \in [0,1]$ and $\sum_{k=1}^{n} \omega_k = 1$

Especially, assume $\omega_k = \frac{1}{n}\ (k = 1,2,......n)$, then F_ω is called an arithmetic average operator for SVNSs.

Similarly, we can define the single valued neutrosophic weighted geometric average operator as follows:

Definition 2.13: Let $A_K(k = 1,2,....n)$ be a SVNS(X).The single valued neutrosophic weighted geometric average operator is defined by

$$G_\omega = (A_1, A_2, ...A_n) = \prod_{k=1}^{n} A_k^{\omega_k}$$

$$\left(\prod_{k=1}^{n} (\mu_{A_k}(x))^{\omega_k}, 1-\prod_{k=1}^{n}(1-w_{A_k}(x))^{\omega_k}, 1-\prod_{k=1}^{n}(1-v_{A_k}(x))^{\omega_k} \right)$$

Where ω_k is the weight of $A_k\,(k=1,2,....n)$, $\omega_k \in [0,1]$ and $\sum_{k=1}^{n}\omega_k = 1$ Especially, assume $\omega_k = \frac{1}{n}\,(k=1,2,......n)$, then G_ω is called an geometric average operator for SVNSs.

3.2. Hexagonal neutrosophic sets

In this section we have extended Hexagonal intuitionstic fuzzy set to Hexagonal neutrosophic sets in which true, indeterminacy and false degrees are included. Also score and accuracy functions for HFNNS are proposed.

As a generalization of a Hexagonal intuitionstic fuzzy set we propose the following definition of a HFNNS

Definition 2.14: Let *X* be a universe of discourse. A Hexagonal neutrosophic set $\tilde{N}$ in X is defined as following. $\tilde{N}_{\tilde{H}} = \{\langle x, T_{\tilde{N}}(x), I_{\tilde{N}}(x), F_{\tilde{N}}(x)\rangle / x \in X\}$ Where $T_{\tilde{N}}(x) \subset [0,1], I_{\tilde{N}}(x) \subset [0,1], F_{\tilde{N}}(x) \subset [0,1]$ are three hexagonal fuzzy numbers such that $T_{\tilde{N}}(x): X \to [0,1]$, $I_{\tilde{N}}(x): X \to [0,1]$, $F_{\tilde{N}}(x): X \to [0,1]$ with the condition $0 \le T_{\tilde{N}}(x) + I_{\tilde{N}}(x) + F_{\tilde{N}}(x) \le 3, x \in X$

Definition 2.15: Assume that $\tilde{A} = (T_{\tilde{A}}(x), I_{\tilde{A}}(x), F_{\tilde{A}}(x))$ *and* $\tilde{B} = (T_{\tilde{B}}(x), I_{\tilde{B}}(x), F_{\tilde{B}}(x))$ be two Hexagonal neutrosophic numbers. Then the following operations rules can be defined as following

$$Let \;\; \tilde{A} = \left[(a_1, b_{1,}, c_{1,}, d_{1,}, e_{1,}, f_{1,}),\; (l_{1,}, m_{1,}, n_{1,}, p_{1,}, q_1, r_{1,}), (u_1, v_1, w_1, x_1, y_1, z_1)\right] and$$
$$\tilde{B} = [(a_2, b_2, c_2, d_2, e_2, f_2),\; (l_2, m_2, n_2, p_2, q_2, r_2), (u_2, v_2, w_2, x_2, y_2, z_2)]$$

be two hexagonal neutrosophic fuzzy numbers then the following operational rules holds.

$$1.\tilde{A} \oplus \tilde{B} = \left\langle \begin{array}{c} (a_1 + a_2 - a_1a_2, b_1 + b_2 - b_1b_2, c_1 + c_2 - c_1c_2, d_1 + d_2 - d_1d_2, e_1 + e_2 - e_1e_2, f_1 + f_2 - f_1f_2), \\ (l_1l_2, m_1m_2, n_1n_2, p_1p_2, q_1q_2, r_1r_2), (u_1u_2, v_1v_2, w_1w_2, x_1x_2, y_1y_2, z_1z_2) \end{array} \right\rangle$$

$$2.\tilde{A} \otimes \tilde{B} = \left\langle \begin{array}{l} (a_1a_2, b_1b_2, c_1c_2, d_1d_2, e_1e_2, f_1f_2), \\ (l_1 + l_2 - l_1l_2, m_1 + m_2 - m_1m_2, n_1 + n_2 - n_1n_2, p_1 + p_2 - p_1p_2, q_1 + q_2 - q_1q_2, r_1 + r_2 - r_1r_2) \\ (u_1 + u_2 - u_1u_2, v_1 + v_2 - v_1v_2, w_1 + w_2 - w_1w_2, x_1 + x_2 - x_1x_2, y_1 + y_2 - y_1y_2, z_1 + z_2 - z_1z_2) \end{array} \right\rangle$$

$$3..\lambda\tilde{A} = \left\langle \begin{array}{l} 1-(1-a_1)^{\lambda}, 1-(1-b_1)^{\lambda}, 1-(1-c_1)^{\lambda}, 1-(1-d_1)^{\lambda}, 1-(1-e_1)^{\lambda}, 1-(1-f_1)^{\lambda}, l_1^{\lambda}, m_1^{\lambda}, n_1^{\lambda}, p_1^{\lambda}, q_1^{\lambda}, r_1^{\lambda}) \\ u_1^{\lambda}, v_1^{\lambda}, w_1^{\lambda}, x_1^{\lambda}, y_1^{\lambda}, z_1^{\lambda}) \end{array} \right\rangle$$

$$4. \;\; (\tilde{A})^{\lambda} \;\; = \left\langle \begin{array}{l} a_1^{\lambda}, b_1^{\lambda}, c_1^{\lambda}, d_1^{\lambda}, e_1^{\lambda}, f_1^{\lambda}), \\ (1-(1-l_1)^{\lambda}, 1-(1-m_1)^{\lambda}, 1-(1-n_1)^{\lambda}, 1-(1-p_1)^{\lambda}, 1-(1-q_1)^{\lambda}, 1-(1-r_1)^{\lambda}) \\ (1-(1-u_1)^{\lambda}, 1-(1-v_1)^{\lambda}, 1-(1-w_1)^{\lambda}, 1-(1-x_1)^{\lambda}, 1-(1-y_1)^{\lambda}, 1-(1-z_1)^{\lambda}), \lambda \geq 0 \end{array} \right\rangle$$

Definition 2.16: Let $\tilde{A} = \langle (a,b,c,d,e,f),(l,m,n,p,q,r),(u,v,w,x,y,z)\rangle$ be a hexagonal neutrosophic fuzzy number. Then the score function of hexagonal neutrosophic number is defined by

$$S(\tilde{A}) = \frac{1}{3}\left[2 + \frac{a+b+c+d+e+f}{6} - \frac{l+m+n+p+q+r}{6} - \frac{u+v+w+x+y+z}{6}\right], S(\tilde{A}) \in [0,1]$$

Definition 2.17 Let $\tilde{A} = \langle (a,b,c,d,e,f),(l,m,n,p,q,r),(u,v,w,x,y,z)\rangle$ be a hexagonal neutrosophic fuzzy number. Then the accuracy function of hexagonal neutrosophic number is defined by

$$H(\tilde{A}) = \frac{a+b+c+d+e+f}{6} - \frac{l+m+n+p+q+r}{6}, H(\tilde{A}) \in [-1,1]$$

Theorem 1: Let $\tilde{A} = \langle (a_1, b_1, c_1, d_1, e_1, f_1), (l_1, m_1, n_1, p_1, q_1, r_1), (u_1, v_1, w_1, x_1, y_1, z_1)\rangle$ be a collection of hexagonal neutrosophic fuzzy numbers, Then their aggregated value using the HNNWAA operator is also a hexagonal neutrosophic number, and then

$$HFNNWA_{\omega}(\tilde{n}_1,\tilde{n}_2,\tilde{n}_3,......\tilde{n}_n) = (\omega_1\tilde{n}_1+\omega_2\tilde{n}_2+\omega_3\tilde{n}_3+.........+\omega_n\tilde{n}_n) = \sum_{i=1}^{n}\omega_i\tilde{n}_i$$

$$= \left\langle \begin{array}{c} \left(1-\prod_{i=1}^{n}(1-a_i)^{\omega_i}, 1-\prod_{i=1}^{n}(1-b_i)^{\omega_i}, 1-\prod_{i=1}^{n}(1-c_i)^{\omega_i}, 1-\prod_{i=1}^{n}(1-d_i)^{\omega_i}, 1-\prod_{i=1}^{n}(1-e_i)^{\omega_i}, 1-\prod_{i=1}^{n}(1-f_i)^{\omega_i}\right) \\ \left(\prod_{i=1}^{n}l_i, \prod_{i=1}^{n}m_i, \prod_{i=1}^{n}n_i, \prod_{i=1}^{n}p_i, \prod_{i=1}^{n}q_i, \prod_{i=1}^{n}r_i\right)\left(\prod_{i=1}^{n}u_i, \prod_{i=1}^{n}v_i, \prod_{i=1}^{n}w_i, \prod_{i=1}^{n}x_i, \prod_{i=1}^{n}y_i, \prod_{i=1}^{n}z_i\right) \end{array} \right\rangle \quad (1)$$

$\omega_i\ (i=1,2,3,.......n), \omega_i \in [0,1]$ is the weight of the i^{th} Hexagonal neutrosophic numbers number $n_i\ (i=1,2,3,.......n), \omega_i \in [0,1], \sum_{i=1}^{n}\omega_i = 1$

Proof: Mathematical induction is used to prove the Equ (A) and the procedure is as follows

(1) When $n = 1$, it is true.

(2) When $n = 2$

$$\omega_1\tilde{n}_1 = \left\langle \begin{array}{c} 1-(1-a_1)^{\omega_1}, (1-(1-b_1)^{\omega_1}, (1-(1-c_1)^{\omega_1}, (1-(1-d_1)^{\omega_1}, (1-(1-e_1)^{\omega_1}, (1-(1-f_1)^{\omega_1}, \\ (l_1)^{\omega_1}, (m_1)^{\omega_1}, (n_1)^{\omega_1}, (p_1)^{\omega_1}, (q_1)^{\omega_1}, (r_1)^{\omega_1}, (u_1)^{\omega_1}, (v_1)^{\omega_1}, (w_1)^{\omega_1}, (x_1)^{\omega_1}, (y_1)^{\omega_1}, (z_1)^{\omega_1} \end{array} \right\rangle$$

$$\omega_2\tilde{n}_2 = \left\langle \begin{array}{c} (1-(1-a_2)^{\omega_2}, 1-(1-b_2)^{\omega_2}, 1-(1-c_2)^{\omega_2}, 1-(1-d_2)^{\omega_2}, 1-(1-e_2)^{\omega_2}, 1-(1-f_2)^{\omega_2},) \\ ((l_2)^{\omega_2}, (m_2)^{\omega_2}, (n_2)^{\omega_2}, (p_2)^{\omega_2}, (q_2)^{\omega_2}, (r_2)^{\omega_2}), ((u_2)^{\omega_2}, (v_2)^{\omega_2}, (w_2)^{\omega_2}, (x_2)^{\omega_2}, (y_2)^{\omega_2}, (z_2)^{\omega_2}) \end{array} \right\rangle$$

Thus using the arithmetic operation (1) in Definition 2.15 we get

$$HFNNWA_{\omega}(\tilde{n}_1\tilde{n}_2) = \omega_1\tilde{n}_1+\omega_2\tilde{n}_2$$

$$= \left\langle \left(\begin{array}{l} [(1-(1-a_1)^{\omega_1} + (1-(1-a_2)^{\omega_2} - (1-(1-a_1)^{\omega_1}(1-(1-a_2)^{\omega_2}, \\ (1-(1-b_1)^{\omega_1} + (1-(1-b_2)^{\omega_2} - (1-(1-b_1)^{\omega_1}(1-(1-b_2)^{\omega_2}, \\ (1-(1-c_1)^{\omega_1} + (1-(1-c_2)^{\omega_2} - (1-(1-c_1)^{\omega_1}(1-(1-c_2)^{\omega_2}, \\ (1-(1-d_1)^{\omega_1} + (1-(1-d_2)^{\omega_2} - (1-(1-d_1)^{\omega_1}(1-(1-d_2)^{\omega_2}, \\ (1-(1-e_1)^{\omega_1} + (1-(1-e_2)^{\omega_2} - (1-(1-e_1)^{\omega_1}(1-(1-e_2)^{\omega_2}, \\ (1-(1-f_1)^{\omega_1} + (1-(1-f_2)^{\omega_2} - (1-(1-f_1)^{\omega_1}(1-(1-f_2)^{\omega_2}], \\ [(l_1)^{\omega_1}(l_2)^{\omega_2}, (m_1)^{\omega_1}(m_2)^{\omega_2}, (n_1)^{\omega_1}(n_2)^{\omega_2}, (p_1)^{\omega_1}(p_2)^{\omega_2}, (q_1)^{\omega_1}(q_2)^{\omega_2}, (r_1)^{\omega_1}(r_2)^{\omega_2}], \\ [(u_1)^{\omega_1}(u_2)^{\omega_2}, (v_1)^{\omega_1}(v_2)^{\omega_2}, (w_1)^{\omega_1}(w_2)^{\omega_2}, (x_1)^{\omega_1}(x_2)^{\omega_2}, (y_1)^{\omega_1}(y_2)^{\omega_2}, (z_1)^{\omega_1}(z_2)^{\omega_2}] \end{array} \right) \right\rangle \quad (2)$$

When $n = k$ equation (1) becomes

$$HNNWAA_{\omega}(\tilde{n}_1, \tilde{n}_2 ....\tilde{n}_k\ ) = \omega_1\tilde{n}_1 + \omega_2\tilde{n}_2 + ...+ \omega_n\tilde{n}_k$$

$$\sum_{j=1}^{k}\omega_j\,\tilde{n}_j = \left\langle \begin{pmatrix} [1-\prod_{i=1}^{k}(1-a_i)^{\omega_i}, 1-\prod_{i=1}^{k}(1-b_i)^{\omega_i}, 1-\prod_{i=1}^{k}(1-c_i)^{\omega_i}, 1-\prod_{i=1}^{k}(1-d_i)^{\omega_i}, 1-\prod_{i=1}^{k}(1-e_i)^{\omega_i}, 1-\prod_{i=1}^{k}(1-f_i)^{\omega_i} \\ [\prod_{i=1}^{k} l_i^{\omega_j}, \prod_{i=1}^{k} m_i^{\omega_j} \prod_{i=1}^{k} n_i^{\omega_j} \prod_{i=1}^{k} p_i^{\omega_j} \prod_{i=1}^{k} q_i^{\omega_j} \prod_{i=1}^{k} r_i^{\omega_j}][\prod_{i=1}^{k} u_i^{\omega_j}, \prod_{i=1}^{k} v_i^{\omega_j} \prod_{i=1}^{k} w_i^{\omega_j} \prod_{i=1}^{k} x_i^{\omega_j} \prod_{i=1}^{k} y_i^{\omega_j} \prod_{i=1}^{k} z_i^{\omega_j}] \end{pmatrix} \right\rangle$$

Then, when $n = k + 1$,by applying Equs (1) and (2) we get

$$HFNNWA_{\omega}(\tilde{n}_1\tilde{n}_2\tilde{n}_3......\tilde{\text{n}}_{k+1}) =$$

$$\left\langle \begin{pmatrix} [1-\prod_{i=1}^{k+1}(1-a_i)^{\omega_i}, 1-\prod_{i=1}^{k+!}(1-b_i)^{\omega_i}, 1-\prod_{i=1}^{k+1}(1-c_i)^{\omega_i}, 1-\prod_{i=1}^{k+1}(1-d_i)^{\omega_i}, 1-\prod_{i=1}^{k+1}(1-e_i)^{\omega_i}, 1-\prod_{i=1}^{k+1}(1-f_i)^{\omega_i}, \\ [\prod_{i=1}^{k+1} l_i^{\omega_j}, \prod_{i=1}^{k+1} m_i^{\omega_j} \prod_{i=1}^{k+1} n_i^{\omega_j} \prod_{i=1}^{k+1} p_i^{\omega_j} \prod_{i=1}^{k+1} q_i^{\omega_j} \prod_{i=1}^{k+1} r_i^{\omega_j}]\ [\prod_{i=1}^{k+1} u_i^{\omega_j}, \prod_{i=1}^{k+1} v_i^{\omega_j} + \prod_{i=1}^{k+1} w_i^{\omega_j} \prod_{i=1}^{k+1} x_i^{\omega_j} \prod_{i=1}^{k+1} y_i^{\omega_j} \prod_{i=1}^{k+1} z_i^{\omega_j}] \end{pmatrix} \right\rangle$$

Namely, when n = k + 1, Eq.(1) is justifiable. Therefore according to the above results we get Equation (1) for any values of n. This completes the proof.

Theorem 2: Let $\tilde{A} = \langle(a_1, b_1, c_1, d_1, e_1, f_1),(l_1, m_1, n_1, p_1, q_1, r_1),(u_1, v_1, w_1, x_1, y_1, z_1)\rangle$ be a collection if hexagonal neutrosophic fuzzy numbers, Then their aggregated value using the HFNNGW operator is also a hexagonal neutrosophic number, and

$$HFNNWG_{\omega}(\tilde{n}_1\tilde{n}_2\tilde{n}_3......\tilde{\text{n}}_n\ ) = (\tilde{n}_1^{\ \omega_1} \otimes \tilde{n}_2^{\ \omega_2} \otimes \tilde{n}_3^{\ \omega_3} + ... \otimes \tilde{n}_n^{\ \omega_n}\ ) = \prod_{i=1}^{n}\tilde{n}_i^{\omega_i}$$

$$= \left\langle \begin{matrix} \left(\prod_{i=1}^{n}a_i, \prod_{i=1}^{n}b_i, \prod_{i=1}^{n}c_i, \prod_{i=1}^{n}d_i, \prod_{i=1}^{n}e_i, \prod_{i=1}^{n}f_i\right), \\ \left(1-\prod_{i=1}^{n}(1-l_i)^{\omega_i}, 1-\prod_{i=1}^{n}(1-m_i)^{\omega_i}, 1-\prod_{i=1}^{n}(1-n_i)^{\omega_i}, 1-\prod_{i=1}^{n}(1-p_i)^{\omega_i}, 1-\prod_{i=1}^{n}(1-q_i)^{\omega_i}, 1-\prod_{i=1}^{n}(1-r_i)^{\omega_i}\right), \\ \left(1-\prod_{i=1}^{n}(1-u_i)^{\omega_i}, 1-\prod_{i=1}^{n}(1-v_i)^{\omega_i}, 1-\prod_{i=1}^{n}(1-w_i)^{\omega_i}, 1-\prod_{i=1}^{n}(1-x_i)^{\omega_i}, 1-\prod_{i=1}^{n}(1-y_i)^{\omega_i}, 1-\prod_{i=1}^{n}(1-z_i)^{\omega_i}\right) \end{matrix} \right\rangle$$

ω_i $(i = 1,2,3,.......n), \omega_i \in [0,1]$ is the weight of the i^{th} Hexagonal neutrosophic numbers number n_i $(i = 1,2,3,.......n), \omega_i \in [0,1], \sum_{i=1}^{n} \omega_i = 1$

This theorem can be proved in the same process as theorem 1.

3.3. Proposed decision making methods by applying HNNWAA and HNNWGA operators

In a problem of multi-attribute decision making, suppose $A_i = \{A_1, A_2, A_3, A_4, A_5\}$ is set of alternatives which satisfies $C_i = \{C_1, C_2, C_3, C_4\}$ *the* set of attributes, $\omega = (\omega_1, \omega_2, \cdots \omega_n)$ is the weighed vector of the attributes, where, $\omega_i, \in [0,1], \sum_{i=1}^{n} \omega_i = 1$. Suppose the characteristic information of alternatives is A_i denoted by hexagonal neutrosphic number $H_{\tilde{N}}(x) = \left\{ \left\langle T_{\tilde{N}}(x), I_{\tilde{N}}(x), F_{\tilde{N}}(x) \right\rangle / x \in X \right\}$ where, $T_{\tilde{A}}(x)$ denotes the true value of alternatives A_i to attribute r_j and $I_{\tilde{A}}(x)$ denotes the indeterminacy value of alternatives A_i to attribute r_j and $F_{\tilde{A}}(x)$ the false value.

We establish a hexagonal neutrosophic decision matrix

Table 3.1
General Decision matrix

	alternative	C_1	C_2	…	C_n
$\tilde{D} = (\tilde{r}_{ij})_{m\times n} =$	A_1	$\tilde{r}_{11}$	$\tilde{r}_{12}$	…	$\tilde{r}_{1n}$
	A_2	$\tilde{r}_{21}$	$\tilde{r}_{22}$	…	$\tilde{r}_{2n}$
	•	•	•	•	•
	A_n	$\tilde{r}_{m1}$	$\tilde{r}_{m2}$	…	$\tilde{r}_{mn}$

$$\tilde{D} = (\tilde{r}_{ij})_{m\times n} = \left\langle (a_{ij}, b_{ij}, c_{ij} d_{ij}, e_{ij}, f_{ij}), (l_{ij}, m_{ij}, n_{ij} p_{ij}, q_{ij}, r_{ij}), (u_{ij}, v_{ij}, w_{ij}, x_{ij}, y_{ij}, z_{ij}) \right\rangle_{m\times n}$$

where $\langle a_{ij}, b_{ij}, c_{ij} d_{ij}, e_{ij}, f_{ij} \rangle \in [0,1]$ indicates the degree that the alternatives A_i is uncertain about the attribute $C_j, \langle l_{ij}, m_{ij}, m_{ij}, p_{ij}, q_{ij}, r_{ij} \rangle \in [0,1]$ indicates the degree that the alternatives A_i satisfies the attribute C_j $\langle u_{ij}, v_{ij}, w_{ij}, x_{ij}, y_{ij}, z_{ij} \rangle \in [0,1]$ indicates the degree that the alternatives A_i does not satisfies the attribute C_j with $0 \leq T_6 + I_6 + F_6 \leq 3$ $for\ i = 1,2,3......m,\ j = 1,2,3......n$ .Based on the HFNNWA and HFNNWG operators we develop a practical approach for solving MADM problems in which the rating of the alternatives over the attributes are expressed with HFNN

The steps of the decision making based on hexagonal neutrosophic fuzzy numbers are as follows:

Step 1: According to the weighted averaging operator or the weighted geometric average operator aggregate all rating values $r_{ij}\,(j = 1,2,......n)$ of the i-th row in the decision making matrix R_{ij}.

$$\tilde{N}_1 = \left\langle (a_1, b_1, c_1, d_1, e_1, f_1), (l_1, m_1, n_1, p_1, q_1, r_1), (u_1, v_1, w_1, x_1, y_1, z_1) \right\rangle$$

$$HFNNWA_{\omega}(\tilde{n}_{i1}, \tilde{n}_{i2}, \tilde{n}_{i3}, ...\tilde{n}_{in})$$

Or by HFNNWG operator as

$$\tilde{N}_1 = \left\langle (a_1, b_1, c_1, d_1, e_1, f_1), (l_1, m_1, n_1, p_1, q_1, r_1), (u_1, v_1, w_1, x_1, y_1, z_1) \right\rangle$$

$$HFNNWG_{\omega}(\tilde{n}_{i1}, \tilde{n}_{i2}, \tilde{n}_{i3}, ...\tilde{n}_{in}))$$

Step 2: Determine the aggregation value S_i corresponding to the alternatives $C_i\,(i = 1,2,3.......m)$ obtained from HFNNWA operator. Calculate the value of the score function $S(\tilde{A}_i)$ and the value of the accuracy function $H(\tilde{A}_i)$ using the formulas of the score function and the accuracy function, where, $i = 1,2,\ldots m$.

Step 3: Rank all the alternatives of $A_i\,(i = 1,2,.....m)$. according to score function and select the best ones

4. NUMERICAL EXAMPLE

A reputed school in the city is interested in selecting books for all grades in their school. The management decided to set a team of 20 teachers were selected and asked to evaluate the books published by Sahara publishers (A_1), White Swam publishers (A_2), Sharathi publishers (A_3) IMAX publishers (A_4) Ramanasagar publishers (A_5) and allot the score looking into the following criteria

(1) Highly qualified authors (C_1)

(2) Conformity with the objectives of curriculum (C_2)

(3) Logical organization (C_3)

(4) Real-life experience (C_4)

The rating of the alternatives $A_i, (i = 1,2,3,4,5)$ with respect to the attributes $C_j, (j = 1,2,3,4)$ are expressed with HFNNs shown in the decision matrix $\tilde{D} = (\tilde{r}_{ij})_{m\times n}$ *Assume* $\omega_1 = 0.25$ $\omega_2 = 0.25$ $\omega_3 = 0.3$ $\omega_4 = 0.2$ the relative weight of all attributes $C_j, (j = 1,2,3,4)$.

Here we apply the proposed aggregation operators HFNNWA and HNFNNWG to find the best books by using the following procedure.

Step 1: The above decision matrix is converted into a singleton neutrosophic fuzzy number using the **HFNNWA** operator and **HFNNWG** and find the values F_{ω_i} $(i = 1,2,3,4,5)$

$$HFNNWA_{\omega} = \left(1 - \prod_{k=1}^{6}(1 - T_{A_k}(x))^{\omega_k}, \prod_{k=1}^{6}(I_{A_k}(x))^{\omega_k}, \prod_{k=1}^{6}(F_{A_k}(x))^{\omega_k}\right)$$

$$HFNNWG_{\omega} = \left(\prod_{k=1}^{6}(T_{A_k}(x))^{\omega_k}, 1 - \prod_{k=1}^{6}(1 - I_{A_k}(x))^{\omega_k}, 1 - \prod_{k=1}^{6}(1 - F_{A_k}(x))^{\omega_k}\right)$$

Table 3.2
Decision matrix

Alternative	*Highly qualified authors* C_1	*Conformity with the objectives of curriculum* C_2	*Logical organization* C_3	*Real-life experience* C_4
A_1	$\left\langle\begin{array}{l}(0.2,0.3,0.4,0.5,0.6,0.7)\\((0.0,0.1,0.2,0.3,0.4,0.5))\\((0.1,0.1,0.1,0.2,0.3,0.3))\end{array}\right\rangle$	$\left\langle\begin{array}{l}(0.0,0.1,0.2,0.3,0.4,0.5)\\((0.0,0.1,0.2,0.3,0.3,0.3))\\((0.2,0.3,0.3,0.3,0.3,0.3))\end{array}\right\rangle$	$\left\langle\begin{array}{l}(0.3,0.4,0.5,0.6,0.7,0.8)\\((0.0,0.1,0.2,0.3,0.4,0.4))\\((0.1,0.1,0.2,0.2,0.3,0.3))\end{array}\right\rangle$	$\left\langle\begin{array}{l}(0.1,0.4,0.4,0.6,0.7,0.9)\\(0.1,0.1,0.1,0.2,0.2,0.3)\\(0.1,0.1,0.1,0.1,0.1,0.1)\end{array}\right\rangle$
A_2	$\left\langle\begin{array}{l}(0.3,0.4,0.5,0.5,0.5,0.5)\\(0.1,0.2,0.3,0.4,0.4,0.5)\\(0.0,0.1,0.1,0.1,0.2,0.2)\end{array}\right\rangle$	$\left\langle\begin{array}{l}(0.2,0.3,0.4,0.5,0.5,0.6)\\(0.0,0.1,0.2,0.3,0.4,0.5)\\(0.0,0.1,0.2,0.3,0.3,0.4)\end{array}\right\rangle$	$\left\langle\begin{array}{l}(0.1,0.1,0.1,0.2,0.2,0.2)\\(0.0,0.1,0.1,0.2,0.2,0.3)\\(0.1,0.1,0.1,0.1,0.1,0.1)\end{array}\right\rangle$	$\left\langle\begin{array}{l}(0.3,0.4,0.5,0.5,0.5,0.6)\\(0.0,0.1,0.1,0.2,0.2,0.2)\\(0.0,0.1,0.1,0.2,0.2,0.3)\end{array}\right\rangle$
A_3	$\left\langle\begin{array}{l}(0.1,0.1,0.3,0.4,0.5,0.6)\\(0.1,0.1,0.1,0.1,0.2,0.2)\\(0.6,0.7,0.8,0.8,0.8,0.8)\end{array}\right\rangle$	$\left\langle\begin{array}{l}(0.1,0.1,0.1,0.2,0.3,0.3)\\(0.1,0.1,0.2,0.3,0.3,0.4)\\0.3,0.4,0.5,0.5,0.6,0.6\end{array}\right\rangle$	$\left\langle\begin{array}{l}(0.2,0.3,0.4,0.5,0.5,0.5)\\(0.1,0.1,0.2,0.3,0.3,0.3)\\(0.1,0.2,0.2,0.3,0.3,0.4)\end{array}\right\rangle$	$\left\langle\begin{array}{l}(0.1,0.2,0.3,0.4,0.4,0.6)\\(0.1,0.1,0.1,0.1,0.1,0.1)\\(0.3,0.4,0.5,0.6,0.6,0.7)\end{array}\right\rangle$
A_4	$\left\langle\begin{array}{l}(0.6,0.6,0.7,0.7,0.7,0.8)\\(0.0,0.1,0.2,0.3,0.4,0.5)\\(0.1,0.1,0.2.,0.3,0.4,0.5)\end{array}\right\rangle$	$\left\langle\begin{array}{l}(0.3,0.4,0.5,0.6,0.7,0.8)\\(0.1,0.1,0.1,0.1,0.2,0.2,)\\(0.0,0.1,0.2,0.2,0.3,0.3)\end{array}\right\rangle$	$\left\langle\begin{array}{l}(0.2,0.3,0.4,0.5,0.6,0.7)\\(0.0,0.1,0.2,0.3,0.4,0.5)\\(0.1,0.2,0.3,0.3,0.4,0.5)\end{array}\right\rangle$	$\left\langle\begin{array}{l}(0.1,0.2,0.2,0.4,0.5,0.6)\\(0.1,0.1,0.1,0.1,0.1,0.1)\\(0.2,0.3,0.4,0.5,0.6,0.7)\end{array}\right\rangle$
A_5	$\left\langle\begin{array}{l}(0.1,0.1,0.2,0.2,0.3,0.3)\\(0.1,0.1,0.1,0.2,0.2,0.2)\\(0.2,0.3,0.4,0.4,0.5,0.6)\end{array}\right\rangle$	$\left\langle\begin{array}{l}(0.2,0.2,0.2,0.2,0.2,0.2)\\(0.0,0.1,0.2,0.3,0.4,0.5)\\(0.1,0.2,0.4,0.3,0.3,0.3)\end{array}\right\rangle$	$\left\langle\begin{array}{l}(0.4,0.5,0.6,0.7,0.8,0.8)\\(0.2,0.3,0.3,0.3,0.3,0.3)\\(0.0,0.2,0.3,0.4,0.4,0.4)\end{array}\right\rangle$	$\left\langle\begin{array}{l}(0.1,0.1,0.1,0.2,0.3,0.3)\\(0.1,0.2,0.3,0.4,0.5,0.6)\\(0.1,0.2,0.3,0.4,0.5,0.6)\end{array}\right\rangle$

Aggregated Hexagonal neutrosophic values for each alternatives using $HFNNWA_{\omega}$ operator

$$F_{\omega_1} = \left\langle\begin{array}{l}(0.1165,0.2614,0.3394,0.4652,0.6909,0.7165),(0,0.0999,0.1740,0.2765,0.3103,0.3598).\\(0.1643,0.1663,0.16630,0.2047,0.2408,0.1732)\end{array}\right\rangle$$

$$F_{\omega_2} = \left\langle\begin{array}{l}(0.2426,0.3354,0.4296,0.4759,0.3844,0.5416),(0.0,0.1549,0.1834,0.2895,0.3247,0.3955)\\(0.0,0.1231,0.1319,0.1782,0.2194,0.2670)\end{array}\right\rangle$$

$$F_{\omega_3} = \left\langle\begin{array}{l}(0..1106,0.1928,0.2379,0.3390,0.4068,0.4884),(0.1000,0.1000,0.1414,0.1732,0.2132,0.2392)\\(0.3309,0.4414,0.5253,0.5673,0.6102,0.6477)\end{array}\right\rangle$$

$$F_{\omega_4} = \left\langle\begin{array}{l}(0.3694,0.4286,0.5328,0.5932,0.6581,0.7608),(0.0,0.1000,0.1319,0.1732,0.2462,0.2752,\\(0.0,0.1335,0.2392,0.2942,0.3866,0.4359)\end{array}\right\rangle$$

$$F_{\omega_5} = \left\langle \begin{matrix} (0.1756, 0.2832, 0.2358, 0.2748, 0.3486, 0.3486), (0.0, 0.1282, 0.1834, 0.2813, 0.3300, 0.3743) \\ (0.0, 0.2258, 0.2780, 0.3565, 0.3986, 0.4366) \end{matrix} \right\rangle$$

Aggregated Hexagonal neutrosophic values for each alternatives using $HFNNWG_{\omega}$ operator

$$F_{\omega_1} = \left\langle \begin{matrix} (0.0, 0.1579, 0.3099, 0.4305, 0.5343, 0.6520), (0.0514, 0.1000, 0.1991, 0.2811, 0.3241, 0.3769) \\ (0.1415, 0.1861, 0.1957, 0.2236, 0.2640, 0.2640) \end{matrix} \right\rangle$$

$$F_{\omega_2} = \left\langle \begin{matrix} (0.2285, 0.2930, 0.3893, 0.4562, 0.4562, 0.5089), (0.0312, 0.1313, 0.1954, 0.3044, 0.3460, 0.4320) \\ (0.0105, 0.1000, 0.1415, 0.2051, 0.2327, 0.2976) \end{matrix} \right\rangle$$

$$F_{\omega_3} = \left\langle \begin{matrix} (0.1071, 0.1282, 0.1989, 0.3099, 0.3898, 0.4464), (0.1000, 0.1000, 0.1515, 0.2063, 0.2339, 0.2797) \\ (0.3932, 0.4985, 0.6019, 0.6244, 0.6564, 0.6806) \end{matrix} \right\rangle$$

$$F_{\omega_4} = \left\langle \begin{matrix} (0.2847, 0.3821, 0.4883, 0.5689, 0.6444, 0.7452), (0.0613, 0.1000, 0.1415, 0.1861, 0.2700, 0.3214) \\ (0.0832, 0.1542, 0.2546, 0.3097, 0.4116, 0.4836) \end{matrix} \right\rangle$$

$$F_{\omega_5} = \left\langle \begin{matrix} (0.1515, 0.1549, 0.1943, 0.2266, 0.2813, 0.2813), (0.0723, 0.1428, 0.2038, 0.2936, 0.3596, 0.4306) \\ (0.1221, 0.2315, 0.2950, 0.3619, 0.3897, 0.4790) \end{matrix} \right\rangle$$

Step 2

The single value neutrosophic values are converted into crisp values using the score function

$S(\tilde{A}_i)(i = 1,2,3,4,5)$ for the collective overall hexagonal neutrosophic number of F_{ω_i} (i = 1, 2, 3, 4, 5) which is shown in table 2

Table 3.3
Score value

Alternatives Ai	*Score value (HFNNWA)*	*Score value (HFNNWG)*	*Ranking*
A_1	0.6718	0.6166	3
A_2	0.6819	0.6517	2
A_3	0.4672	0.4258	5
A_4	0.7372	0.6880	1
A_5	0.5520	0.4856	4

Table 3.4
Ranking of the alternatives

Aggregation operator	*Ranking order*
HFNNWA	$A_4 > A_2 > A_1 > A_5 > A_3$
HFNNWG	$A_4 > A_2 > A_1 > A_5 > A_3$

The management is pleased to implement the books published by the publisher A_4 which ranked on top.

5. CONCLUSION

This paper proposes hexagonal fuzzy neutrosophic number and arithmetic operations on HFNNs as an extension of HIFN. We proposed HFNNWA operator and HFNNWG operators to aggregate the decision making matrix. Also score and accuracy functions were defined and using the score function the aggregated value was made in to a crisp value to make ranking alternatives easy and simple. At last an illustration was given to show the application of the proposed decision making method. In future research we have decided to investigate the application of these operators to study the stress upon students in classroom learning environments.

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