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Precise Positioning of GPS Receivers using Least Squares Method

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Abstract: Precise positioning measurements of Global positioning systems (GPS) have revolutionized scientific research in the area of geodynamics. Nowadays, it aids in the study of the crustal deformations caused by Earthquakes, the unavoidable natural disasters. Calibration of the GPS data over fault lines, epoch by epoch results in precise positioning of the receiver. Least squares method is implemented in the calculation of the receiver coordinates obtained from the pseudoranges for every epoch. Application of this method has given a good approximation of the receiver coordinates at a given location.

Keywords: Signal processing algorithms, GPS receiver coordinates, Pseudoranges.

1. INTRODUCTION

Inaccurate GPS receiver position coordinates are caused by errors in satellite ephemerides, ionosopheric modeling, calibration of local equipment etc. These errors are minimized by calibration of the GPS receiver epoch by epoch [1]. Baseline estimation using GPS receivers is useful for the analysis of the crustal deformation caused by earthquakes. Instantaneous processing of GPS data epoch by epoch reduces the error observed in multi-epoch processing.

In the conventional processing techniques (batch processing and 24 hour data) the receiver has to re-initialize carrier phase ambiguities occurring due loss of lock. The methods provide inaccurate position coordinates over the baselines [2], [3], [4]. Epoch by epoch processing resolves the complexity arising due to loss of lock, initialization of the carrier phase ambiguities as it considers the first available time information [5], [6], [7].

GPS receivers record the data at regular intervals of time for example every 30 seconds. The receiver clock time at which the data is recorded is the measurement time given by T and also the observation file is recorded with the receiver measurement time. Thus the actual observation time of the satellite is

$$\rho^s = (T_1 - T_1^s)c \tag{1}$$

where, 'T_1^S' is the satellite clock time when the signal is transmitted and 'c' is the speed of light in vacuum. The receiver and the satellite clocks have bias. These clock biases have to added with the measured and satellite times. The corrected measured and satellite times are given as

$$T = t - \tau \tag{2}$$

$$T_1^s = t^s - \tau^s \tag{3}$$

where, 'τ' and 'τ^s' are the receiver and satellite clock biases. Thus pseudorange '$\rho^s(t)$' is given as

$$\rho_1^s(t) = ((t + \tau) - (t^s + \tau^s))c$$

$$\rho_1^s(t) = (t - t^s)c + c\tau - c\tau^s$$

$$\rho_1^s(t) = \rho^s(t, t^s) + c\tau - c\tau^s \tag{4}$$

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where, $\rho^s(t, t^s)$ is the range from receiver to the satellite. In the above equation we assume that the speed of light in atmosphere is 'c' ignoring the theory of relativity. If we know the satellite position in the orbit (x^s, y^s, z^s) and the satellite clock bias τ^s, then the pseudorange is given as

$$\rho^s(t, t^s) = \sqrt{V_1 + V_2 + V_3} \tag{5}$$

where, $V_1 = (x^s(t^s) - x(t))^2$, $V_2 = (y^s(t^s) - y(t))^2$, $V_3 = (z^s(t^s) - z(t))^2$

The satellite coordinates can be calculated from the navigation message information transmitted from the GPS satellite. Thus we have four unknown values i.e., receiver coordinates and the receiver clock bias. The maximum satellite range from the time the signal has been transmitted is 60 meters i.e., 0.07 seconds lagging when it reaches the receiver. If we know the receiver clock bias we can compute the transmit time. The receiver clock bias is usually few milliseconds i.e., relatively 50 meters when S/A is switched on. Thus it can be cautiously ignored in error computation of pseudoranges of different satellites. This error can be corrected by using more precise carrier phase observables [8]. Let us assume four different satellites with pseudoranges ρ_1, ρ_2, ρ_3, ρ_4 given as

$$\rho_1 = ((x_{s_1} - x_r)^2 + (y_{s_1} - y_r)^2 + (z_{s_1} - z_r)^{1/2} + c\tau - c\tau_{s_1} \tag{6}$$

$$\rho_1 = ((x_{s_2} - x_r)^2 + (y_{s_2} - y_r)^2 + (z_{s_2} - z_r)^{1/2} + c\tau - c\tau_{s_2} \tag{7}$$

$$\rho_1 = ((x_{s_3} - x_r)^2 + (y_{s_3} - y_r)^2 + (z_{s_3} - z_r)^{1/2} + c\tau - c\tau_{s_3} \tag{8}$$

$$\rho_1 = ((x_{s_4} - x_r)^2 + (y_{s_4} - y_r)^2 + (z_{s_4} - z_r)^{1/2} + c\tau - c\tau_{s_4} \tag{9}$$

In the present work Least squares method is implemented to find out the precise receiver coordinates epoch by epoch. This method is easier to implement on large amount of data and get a good prediction of the input variables. Least square method is simpler to analyze mathematically and its solutions can easily be interpreted. It works in limited number of points and reduces the processing time, prediction time and computer memory. The implementation of least squares method at every epoch results in the precise estimation of the receiver coordinates over fault lines aiding in the study of geodynamics caused by earthquakes.

2. METHODOLOGY

The accurate estimation of the receiver coordinates is solved by linerising the pseudoranges. Then the method of least squares is analysis is implemented. Let us assume the physical observation ρ_o is the sum of modeled observations ρ_m, plus an error term given as

$$\rho_o = \rho_m + \text{noise} \tag{10}$$

$$\rho_o = \rho(x_s, y_s, z_s, \tau_s) + v \tag{11}$$

Now, by considering the initial values $(x_{s_o}, y_{s_o}, z_{s_o}, \tau_{s_o})$ we expand the above equation using Taylor's theorem and ignore the second and higher order terms given as

$$\rho(x, y, z, t) \cong \rho(x_{s_o}, y_{s_o}, z_{s_o}, \tau_{s_o}) + (x - x_{s_o})\frac{\partial\rho}{\partial x} + (y - y_{s_o})\frac{\partial\rho}{\partial y} + (z - z_{s_o})\frac{\partial\rho}{\partial z} + (\tau - \tau_{s_o})\frac{\partial\rho}{\partial\tau}$$

$$= \rho_{\text{computed}} + \frac{\partial\rho}{\partial x}\Delta x + \frac{\partial\rho}{\partial y}\Delta y + \frac{\partial\rho}{\partial z}\Delta z + \frac{\partial\rho}{\partial\tau}\Delta\tau \tag{12}$$

The partial derivatives in the above equation are computed using the initial values. The residual pseudorange is given as the difference between the observed and provisional parameter values as

$$\Delta\rho = \rho_o - \rho_{\text{computed}} \quad (13)$$

$$\Delta\rho = \frac{\partial\rho_c}{\partial x}\Delta x + \frac{\partial\rho_c}{\partial y}\Delta y + \frac{\partial\rho_c}{\partial z}\Delta z + \frac{\partial\rho_c}{\partial\tau}\Delta\tau + v \tag{14}$$

This can be written in the form

$$\Delta\rho = \begin{pmatrix} \frac{\partial\rho_c}{\partial x} & \frac{\partial\rho_c}{\partial y} & \frac{\partial\rho_c}{\partial z} & \frac{\partial\rho_c}{\partial\tau} \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta\tau \end{pmatrix} + v \tag{15}$$

Such approximation is drawn for the satellites in view. The above equation can also be written as

$$b_1 = \mathrm{A}_1 x + v_1 \tag{16}$$

which explicit a linear relation between the residual observations and the unknown correction to the parameters x. The design matrix in the above equation for four satellites is given as

$$\mathrm{A} = \begin{pmatrix} \frac{x_0 - x_{s_1}}{\rho} & \frac{y_0 - y_{s_1}}{\rho} & \frac{z_0 - z_{s_1}}{\rho} & c \\ \frac{x_0 - x_{s_2}}{\rho} & \frac{y_0 - y_{s_2}}{\rho} & \frac{z_0 - z_{s_2}}{\rho} & c \\ \frac{x_0 - x_{s_3}}{\rho} & \frac{y_0 - y_{s_3}}{\rho} & \frac{z_0 - z_{s_3}}{\rho} & c \\ \frac{x_0 - x_{s_4}}{\rho} & \frac{y_0 - y_{s_4}}{\rho} & \frac{z_0 - z_{s_4}}{\rho} & c \end{pmatrix}$$

The least square solution:

The solution for the linearised observation is considered as $\hat{x}$. Thus by using the linearised equation given above we can write

$$\hat{v} = b_1 - \mathrm{A}\hat{x} \tag{17}$$

where the estimated residual is given as difference of the original observations and the estimated model observations. Thus the solution of the least squares can be written as

$$g(x) \equiv \sum_{i=1}^{4} v_i^2 = v^{\mathrm{T}} v = (b_1 - \mathrm{A}x)^{\mathrm{T}}(b_1 - \mathrm{A}x) \tag{18}$$

From the above equation we are minimizing the estimated residuals. The following equations describe the application of the above method.

$$\delta g(\hat{x}) = 0 \tag{19}$$

$$\delta\{(b_1 - \mathrm{A}_1\hat{x})^{\mathrm{T}}(b_1 - \mathrm{A}_1\hat{x})\} = 0 \tag{20}$$

$$\delta(b_1 - \mathrm{A}_1\hat{x})^{\mathrm{T}}(b_1 - \mathrm{A}_1\hat{x}) + (b_1 - \mathrm{A}_1\hat{x})^{\mathrm{T}}\delta(b_1 - \mathrm{A}_1\hat{x}) = 0 \tag{21}$$

$$(-\mathrm{A}_1\delta x)^{\mathrm{T}}(b_1 - \mathrm{A}_1\hat{x}) + (b_1 - \mathrm{A}_1\hat{x})^{\mathrm{T}}(-\mathrm{A}_1\delta\hat{x}) = 0 \tag{22}$$

$$(-2A_1\delta x)^T (b_1 - A_1\hat{x}) = 0 \tag{23}$$

$$(\delta x^T A_1^T)(b_1 - A_1\hat{x}) = 0 \tag{24}$$

$$\delta x^T (A_1^T b_1 - A_1^T A_1\hat{x}) = 0 \tag{25}$$

$$A_1^T A_1\hat{x} = A_1^T b_1 \tag{26}$$

Equation (26) represents normal equations. The solution is given by

$$\hat{x} = \text{inverse}((A_1^T A_1))A_1^T b \tag{27}$$

In the present case data from the ohiosat is considered for analysis. For a single epoch the algorithm is implemented consisting of five satellites. The pseudoranges are taken from the RINEX observation file collected on 30th March 1996.

3. RESULTS AND DISCUSSION

In the present work we have implemented the least square method for precise positioning of the GPS receiver epoch by epoch. Calibration of receiver position epoch by epoch will lead to a better estimation of the receiver coordinates. The receiver position for a particular epoch is represented in the Figure 1 given below.

It is shown that the instantaneous positioning provides good results than that of batch processed data. The studies carried out over the baselines situated in the earthquake prone area will aid in the analysis of coseismic, preseismic and crustal motions in that region. Single epoch analysis has provided a trade –off between the computation times, precision when compared with the conventional techniques like static and kinematic processes.

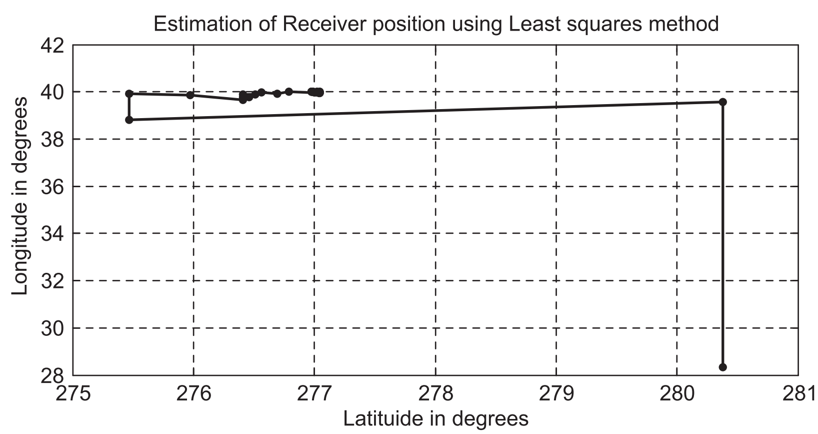

Figure 1: Estimation of receiver position using least squares method

Epoch time smoothing has to be done before the calculation of the accurate receiver coordinates as this analysis overcomes the problems associated with carrier phase ambiguities.

Calibration of GPS receivers by the implementation of conventional techniques, initializes the integer cycle phase ambiguities at the first instant of its occurrence. Once it is fixed it will not be changed further even though there are other incidents of their occurrence. In addition to the provision of providing accurate positioning, the method of instantaneous processing of GPS data also resolves integer-cycle phase ambiguities before proceeding further. Thus in this method the receiver is reinitialized for every occurrence of loss of lock. This re-initialization introduces delay of 30-40 seconds. In this process one the receiver comes out of this problem and the hardware reacquires the GPS satellites the software will fully reclaim on the first available epoch.

4. CONCLUSIONS

The least square method presented for the calibration of instantaneous receiver coordinates has given more accurate results compared to that of the batch processing methods. This method avoids the risk of cleaning the data, and re-initialization of carrier phase ambiguities at time of loss of lock. It also provides better parameter estimates at high frequencies and reduces the effect of flicker noise seen in the coordinate estimation caused due to atmospheric delays for longer baselines.

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References

1. Genrich, J., and Y. Bock, rapid resolution of crustal motion at short ranges with the Global Positioning System, J. Geophys. Res., 97, 3261-3269, 1992.
2. Genrich, J.F., Y. Bock, and R. Mason, Crustal deformation across the Imperial fault: Results from kinematic GPS surveys and trilateration of a densely-spaced, small aperture network, J. Geophys. Res., 102, 4985-5004, 1997.
3. Blewitt, G., An automatic editing algorithm for GPS data, Geophgs. Res. Left., 17, 199-202, 1990.
4. Blewitt, G., Carrier phase ambiguity resolution for the GlobM Positioning System applied to geodetic baselines up to 2000 km, J. Geophys. Res., 9•, 10,187-10,203, 1989.
5. Dong, D., and Y. Bock, Global Positioning System network analysis with phase ambiguity resolution applied to crustal deformation studies in California, J. Geophys. Res., 9J, 3949-3966, 1989.
6. Bock, Y., and S. Williams, Integrated satellite interferometry in southern California, Eos Trans. A GU, 78, 293, 2s-a00, 1997.
7. M. Akhoondzadeh "Anomalous TEC variations associated with the powerful Tohoku earthquake of 11 March 2011", Nat. Hazards Earth Syst. Sci., 12, 1453–1462, 2012 © Author(s) 2012. CC Attribution 3.0 License.
8. Basics of the GPS Technique: Observation Equations, "Geoffrey Blewitt", Department of Geomatics, University of Newcastle, Newcastle upon Tyne, NE1 7RU, United Kingdom.