A 2 Dimensional Direction of Arrival Estimation with Pair Matching Algorithm for Adaptive Array Antenna

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ABSTRACT

It is known that smart antenna is adequate for 3G/ 4G mobile communication. Smart antenna is used in mobile communication to describe adaptive processes designed to improve the capacity and bandwidth[1]. It employs adaptive beamforming techniques. DOA (direction of arrival) estimation is an essential part of smart antenna. DOA estimation parameters are used for source localization and it provides the desired signal locations.

In this paper we consider a L shaped antenna array. L shaped array provides high estimation accuracy, low computational burden, and easy analysis. The most popular techniques in DOA estimation are MUSIC and ESPRIT. These algorithms are based on Eigen value decomposition (EVD) of cross spectral matrix. The computational complexities are very high and costly. The no of sources and antenna elements are large.

This paper will employ a novel algorithm which is implemented on the L shaped antenna array. The computational method is very simple. The conventional Pair matching method is adapted. We have compared our result with MUSIC and ESPRIT.

Key words: DOA, L shaped array, MUSIC, ESPRIT, Pair matching.

1. INTRODUCTION

The problem is to estimate elevation and azimuth angle of received signals. It has been shown an L shaped antenna array provides a better performance than any other shaped array (ULA, planner array, circular array, and parallel shaped array. For accurate and fast estimation of the direction of arrival of the transmitted signals a L shaped array antennas is considered. The detection and estimation of parameters of multiple waves are discussed.

Many DOA estimation algorithms have been developed using beam former methods like MUSIC [2], ESPRIT [3, 4]. A lot of work has been done on ULA and the 1D DOA azimuth angle is estimated [8-11]. For 2 D DOA estimation any type of planner array is required.

<table>
<thead>
<tr>
<th>Array Type</th>
<th>CRB cos(αₙ)</th>
<th>cos(βₙ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Octagon array</td>
<td>57/6N³</td>
<td></td>
</tr>
<tr>
<td>L shaped Array</td>
<td>60/6N³</td>
<td></td>
</tr>
<tr>
<td>Cross Array</td>
<td>96/6N³</td>
<td></td>
</tr>
<tr>
<td>Square Array</td>
<td>96/6N³</td>
<td></td>
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<tr>
<td>Right Triangle Array</td>
<td>108/6N³</td>
<td></td>
</tr>
<tr>
<td>Generalized cross Array</td>
<td>192/6N³</td>
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</table>

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In this paper we have presented MUSIC and ESPRIT. We propose a new algorithm 2 D DOA estimation. We compare our results with MUSIC and ESPRIT. A L shaped array consists of two ULA connected orthogonally at one end. The CRB (Cramer-Rao Bound) estimation of different array are given in table 1. [12]

$$\delta = 2 \text{SNR} 1 (2 \pi c/\lambda)^2, \text{SNR} 1 = \frac{|a|^2}{2 \delta^2}$$ where $2 \delta^2$ is the variance of the white noise. As the array are symmetry in nature $\text{CRB}(\alpha_i) = \text{CRB}(\beta_1)$, $\text{CRB}(\alpha_i) = \text{CRB} \cos(\alpha_i)/\sin^2(\alpha_i)$ and $\text{CRB}(\beta_1) = \text{CRB} \cos(\beta_1)/\sin^2(\beta_i)$.

From the table only octagon array is 5% less CRB than L shaped array. CRB of L shaped array is 37% smaller than cross array. L shaped array has higher accuracy level than cross arrays and many other simple arrays.

2. DATA MODEL

Two uniform linear orthogonal arrays considered and these form a L shaped array configuration. Each array contains M no of antenna elements and the spacing between each elements is d. Let us say there are K narrow band far field sources impinging on the antenna array from different directions. The signal wavelength be $\lambda$ and the $k^{th}$ source has elevation angle be $\theta_k$ and azimuth angle be $\phi_k$, $k = 1, 2, \ldots, K$. The angle of $k^{th}$ source with respect to X axes is represented in the fig 1.

![Figure 1: L shaped array configuration for 2-D AOA estimation.](image)

The observed signal along X axes sub array and Z axes sub array be

$$x(t) = A(\phi) s(t) + n_x(t)$$

$$z(t) = A(\theta) s(t) + n_z(t)$$

The matrices and the vectors in (1) and (2) have the following forms.

$$z(t) = [z_1(t), z_2(t), \ldots, z_M(t)]^T$$

$$x(t) = [x_1(t), x_2(t), \ldots, x_M(t)]^T$$

$$A(\theta) = [a(\theta_1), a(\theta_2), \ldots, a(\theta_M)]$$

$$A(\phi) = [a(\phi_1), a(\phi_2), \ldots, a(\phi_M)]$$
A 2 Dimensional Direction of Arrival Estimation with Pair Matching Algorithm...

\[ A(\theta_k) = [1, e^{j\phi_k}, \ldots, e^{j(M-1)\phi_k}]^T \]
\[ A(\phi_k) = [1, e^{j\theta_k}, \ldots, e^{j(M-1)\theta_k}]^T \]

\[ \phi_k = \pi \cos \theta_k, \beta_k = \pi \cos \phi_k \]

\[ s(t) = [s_1(t), s_2(t), \ldots, s_K(t)]^T \]
\[ n_z(t) = [n_{z1}(t), n_{z2}(t), \ldots, n_{zM}(t)]^T \]
\[ n_x(t) = [n_{x1}(t), n_{x2}(t), \ldots, n_{XM}(t)]^T \]

The subscripts T denote transpose. \( Z_m(t) \) and \( x_m(t) \) denote \( m^{th} \) element of received data in \( Z \) axes and \( X \) axes sub array respectively. \( S(t) \) is the \( K \times 1 \) vector of source signal. \( n_z(t) \) and \( n_x(t) \) are additive noise and are independent of signal samples. The elements of \( \{ n_z(t) \text{ and } n_x(t) \} \) are white Gaussian random processes with zero mean and variance \( \sigma^2_n \). \( A(\theta)(a(\theta_k)) \) and \( A(\phi)(a(\phi_k)) \) are the \( M \times K \) \( (M \times 1) \) array response matrices in \( Z \) axes and \( X \) axes sub array.

3. DOA ALGORITHMS

3.1. MUSIC

MUSIC is the most defacto algorithm to estimate multiple source parameters like elevation angle, azimuth angle, range, polarization etc. MUSIC requires a priori knowledge of spatial background noise and interferences. It says The desired signal array response is orthogonal to noise subspace \([2A]\). The signal and noise subspace are identified by using Eigen value decomposition of the received signal covariance matrix. MUSIC spatial spectrum is calculated and DOA is estimated. In general an array is set in the region of interest in the DOA space. The region of \( \theta \) is extracted from the region of \( A(\theta) \). \( A(\theta) \) is the array response vector. The subspace estimation is achieved by eigen decomposition of the auto-covariance matrix of the received data \( R_{xx} \).

It is assumed that the spatial whiteness \( E \{n_z(t)n_z^H(t)\} = \sigma_n^2 I \). The eigen value

\[ \lambda_n = \lambda_1 > \lambda_2 > \ldots > \lambda_k > \lambda_{k+1} = \sigma_n^2 \]

The eigenvectors \( e_n \in \mathbb{C}^N, n = 1, 2, \ldots N \), for \( R_{xx} \)

\[ E = [E_s, E_n] \]

\[ = E_s \Lambda_s E_s^H + \sigma_n^2 E_n E_n^H = E_s \Lambda_s E_s^H + \sigma_n^2 I \]

\[ = [E_s, E_n], \]
\[ E_s = [e_1, e_2, \ldots e_k], \]
\[ E_n = [e_{k+1}, e_{k+2}, \ldots e_N], \]
\[ \Lambda = \text{diag} \{ \lambda_1, \lambda_2, \ldots, \lambda_N \}, \]
\[ \Lambda_s = \text{diag} \{ \lambda_1, \lambda_2, \ldots, \lambda_k \}, \]
\[ \Lambda_n = \text{diag} \{ \lambda_{k+1}, \lambda_{k+2}, \ldots, \lambda_N \}, \text{and} \]
\[ \Lambda_s = \Lambda_s - \sigma_n^2 I. \]

The eigen vector \( E = [E_s, E_n] \) can be assumed to form an orthogonal basis. The span of \( K \) vector \( E_s \) defines the signal subspace and \( E_n \) defines the noise subspace. After determining the subspaces the DOA of the desired signal can be calculated through MUSIC algorithms \([3]\)

\[ P_{\text{MUSIC}}(\theta) = \frac{a^H(\theta) a(\theta)}{a^H(\theta) E_n E_n^H a(\theta)} \]
3.2. ESPRIT

Estimation of Signal Parameters via Rotational Invariance Techniques are as follows. An antenna comprise of two identical sub arrays. Some antenna array elements may be member of both sub arrays\cite{4}. Let an array contains M elements and m elements which are member of both sub array (so that M < 2m). The individual elements of sub array can have arbitrary polarization, directional gain, phase response.

Let “d” signals impinging onto the array. Let x1(t) and x2(t) are signal received by the two sub arrays and let the received signals are corrupted by additive noise n1(t) and n2(t). Each of the sub-arrays has m elements. The elements are separated by a fixed displacement vector D. The received signals may be expressed as

\[ x_1(t) = [a(\mu_1),...a(\mu_d)]s(t) + n_1(t) = A_1\Theta s(t) + n_1(t) \]  
\[ x_2(t) = [a(\mu_1)e^{j\phi_1},...a(\mu_d)e^{j\phi_d}]s(t) + n_2(t) = A_2\Theta s(t) + n_2(t) \]  

x_1(t) and x_2(t) are the m x 1 vectors represents received data of 1st and 2nd sub array. n_1(t) and n_2(t) are m x 1 noise vectors. A_1\Theta and A_2\Theta belongs to \( C^{m \times k} \). This indicates the manifold of each sub array is unitary diagonal matrix

Let J1 and J2 represent the M x m selection matrix .

\[ J_1 = \begin{bmatrix} 0_M \times (m - M) : I_m \end{bmatrix} \]  
\[ J_2 = \begin{bmatrix} 0_M \times (m - M) : I_m \end{bmatrix} \]  

I_m is the M x M identity matrix and 0_M \times (m-M) is the M \times (m-M) matrix of zeros. The two identical sub arrays satisfies

\[ \theta_k = \sin^{-1}\left\{ \frac{\arg\{\}D}{2\pi} \right\}, i = 0, 1...K \]  
\[ JA(\Theta) = \begin{bmatrix} J_1 \\ J_2 \end{bmatrix} A(\Theta) = \begin{bmatrix} A_1(\Theta) \\ A_2(\Theta) \Phi \end{bmatrix} \]  

Where \( \Phi \) is the diagonal matrix and

\[ \Phi_i = \exp\{-j\beta_i^T.D\}, i = 1, 2..K \]  

\( \beta_i \) = vector wave number of incident plane from ith narrow band source

D = vector displacement between two sub array.

\( E_s \) = eigenvector corresponding to K largest eigen values of received signal

\[ E_s \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} A_1(\Theta) \\ A_2(\Theta) \Phi \end{bmatrix}^T \]
T is full rank matrix and \( T \in \mathbb{C}^{K \times K} \).

Solving the equation (11) we can get

\[
E_2 = E_1 T^{-1} \Phi T = E_1 \Psi
\]  

(12)

Where \( \Psi = T^{-1} \Phi T \) or \( \Phi = T \Psi T^{-1} \). So eigen values of \( \Psi \) must be equal to diagonal elements of \( \Phi \). This is the fundamental relation in the properties of ESPRIT. If \( M > K \) and \( D = |D| < \frac{\lambda}{2} \). From the eigen values of operator, \( \Psi \), \( E_1 \) and \( E_2 \), DOA can be determined.

\[
\theta_k = \sin^{-1} \left\{ \frac{\arg \{ \Psi_i \}}{2 \pi \frac{\lambda}{D}} \right\}, i=0,1,...,K
\]  

(13)

\( \Psi_i \) is the each eigen value of \( \Psi \) matrix.

3. DIRECTION OF ARRIVAL ESTIMATION BY ADAPTIVE ARRAY ANTENNA PLANE

A new technique for 2 D DOA estimation is proposed. Consider a L shaped array has antenna elements in X and Z axes. Each ULA contains N no of elements and the distance between each element is \( d \). A narrow band signal \( \{ S_k(t) \}_{k=1}^K \) impinge on the array in an elevation angle \( \{ \theta_k \}_{k=1}^K \) and azimuth angle \( \{ \phi_k \}_{k=1}^K \). Both elevation angle and azimuth angle can be found out independent. It is presumed both ULA are synchronized. The pair matching method can be exploited independently. The baseband signals of the \( t \)th snapshot of the array along Z axes is expressed as

\[
z(t) = \sum_{k=1}^{K} a(\theta_k) S_k(t) + n_z(t)
\]  

(14)

\( a(\theta_k) \) is called steering vector.

\[
a(\theta_k) = [a_1(\theta_k) \cdots a_m(\theta_k)]^T
\]  

(15)

The signal component arriving on nth antenna element. For an adaptive antenna system, if \( p \) users transmit signals from different locations, and each user’s signal arrives at the array through multiple paths. Let \( L \) denote the number of multipath components of \( i \)th user. We have \( \sum_{i=1}^{p} L_{Mi} = p \). Let’s further assume that all of the multi path components for a particular user arrive within a time window which is much less than the channel symbol period for that user, then the input data vector could be expressed as-

\[
x(t) = \sum_{i=1}^{p} \sum_{k=1}^{L_{Mi}} \alpha_{i,k} a(\theta_{i,k}) S_i(t) \cdot n(t)
\]  

(14)

or we can write

\[
x(t) = \sum_{i=1}^{p} G_i S_i(t) + n(t)
\]  

(15)

where \( \theta_{i,k} \) is the DOA of the k-th multipath component for the i-th user, \( a(\theta_{i,k}) \) is the steering vector corresponding to \( \theta_{i,k} \), \( \alpha_{i,k} \) is the complex amplitude of the k-th multipath component for the i-th user, and \( G_i \) is the spatial signature for the i-th user and is given by
The signal component arriving on \( n \)th antenna element at a particular instance of time is given by

\[
X_n = A \exp(j 2 \pi n dsin \theta / \lambda)
\]

(17)

\[
Y_n = A \exp(j 2 \pi n dsin \theta / \lambda)
\]

(18)

Where \( A \) = complex amplitude of the signal, \( \phi \) = Direction of Arrival (DOA) of the signal (Azimuth Angle) (unknown), \( \theta \) = Direction of Arrival (DOA) of the signal (Elevation Angle) (unknown), \( d \) = spacing between antenna elements and \( \lambda \) = wavelength.

Now one can view (4) & (5) as-

\[
X_n = A \exp[j 2 \pi f (n dsin \theta \cos / c)]
\]

(19)

\[
Y_n = A \exp[j 2 \pi f (n dsin \theta \sin / c)]
\]

(20)

Where \( f \) = frequency of the signal and \( c \) = velocity of wave.

Now if we mechanically steer the antenna plane by \( \delta \phi \) & \( \delta \theta \), then (19) & (20) becomes –

\[
X_1^n = A \exp[j 2 \pi f (n dsin \theta \cos(\phi + \delta \phi) / c)]
\]

(21)

\[
Y_1^n = A \exp[j 2 \pi f (n dsin \theta \sin(\phi + \delta \phi) / c)]
\]

(22)

\[
X_2^n = A \exp[j 2 \pi f (n dsin(\theta + \delta \theta) \cos \phi / c)]
\]

(23)

\[
Y_2^n = A \exp[j 2 \pi f (n dsin(\theta + \delta \theta) \sin \phi / c)]
\]

(24)

Now taking the frequencies (which can be known by seeing the spectra of the signal) of the signal from (19) and (21), and taking their ratio one could get-

\[
\frac{\text{frequency} \rightarrow X_n}{\text{frequency} \rightarrow X_1^n} = \frac{\cos \phi}{\cos(\phi + \delta \phi)} = \frac{1}{k} \quad (k \text{ is known})
\]

Hence

\[
\phi = \tan^{-1}\left[\frac{\cos \delta \phi - k}{\sin \delta \phi}\right]
\]

(25)

And from (20) & (24), we could get

\[
\frac{\text{frequency} \rightarrow Y_n}{\text{frequency} \rightarrow Y_1^n} = \frac{\sin \theta}{\sin(\theta + \delta \theta)} = \frac{1}{k} \quad \theta = \cot^{-1}\left[\frac{k - \sin \theta}{\cos \delta \theta}\right]
\]

(26)

Now using the simple relation given in (25) & (26) one can determine the unknown DOA (\( \theta \) & \( \phi \)) of all incoming signal impinging on the array with suitable algorithm based on (19), (20), (21), (22), (23), (24), (25) and (26).

### 3.1. Pair Matching

The pair matching algorithm utilize cross correlation matrix of received signals on both ULAs. The pair matching done separately for 2D estimation.

The baseband signal of \( t \)-th snapshot of the array output measured along Z axis is expressed as [13].

The observed signal along X axes sub array and Z axes sub array be
The cross correlation matrices $R_{zx}$ between $z(t)$ and $x(t)$.

$$R_{zx} = E\{z(t)x^H(t)\} \tag{27}$$

$$= \sum a(\theta_k)E\{s_k(t)s_k^H\}(\phi_k) + E\{n_z(t)n_x^H(t)\} \tag{28}$$

The matrix

$$A(\theta) = [a(\theta_1), \ldots, a(\theta_K)]$$

$$A(\phi) = [a(\phi_1), \ldots, a(\phi_K)]$$

$S = \text{diag}\{r_1, \ldots, r_K\}$, is the power of $k$th signal.

$$N_{zx} = E\{n_z(t), \ldots, n_x^H(t)\}$$

The noise vector $n_z(t)$ and $n_x^H(t)$ are independent and uncorrelated. So $N_{zx} = 0$

The cross-correlation matrix

$$R_{zx} = A(\theta)SA^H(\phi) \tag{29}$$

Whose $(p, q)$ element is

$$[R_{zx}]_{p,q} = \sum_{k=1}^{K} r_k \exp[-j\mu\{(p-1)\cos\theta_k - (q-1)\cos\phi_k\}]$$

Where $\mu \equiv 2\pi d / \lambda$.

Based on the diagonal elements of $R_{zx}$

$$r_{zx} = \left(\sum_{k=1}^{K} r_k, \sum_{k=1}^{K} r_k e^{-j\mu w_1}, \ldots, \sum_{k=1}^{K} r_k e^{-j\mu(M-1)w_1}\right)^T$$

Where

$$w_k = \cos\theta_k - \cos\phi_k, \ k = 1, \ldots, K$$

The relation between elevation angle and azimuth angle emerge in vector $r_{zx}$.

$R_{cc}$ is the toeplitz matrix and its first column and row are $r_{zx}$ and $r_{zx}^H$.

$$R_{cc} = \begin{bmatrix} r_{zx}(1) & r_{zx}^*(2) & \cdots & r_{zx}^*(M) \\ r_{zx}(1) & r_{zx}(1) & \cdots & \vdots \\ \vdots & \vdots & \ddots & r_{zx}^*(2) \\ r_{zx}(M) & \cdots & \cdots & r_{zx}(1) \end{bmatrix} \tag{30}$$

The angle of arrival techniques using covariance matrix are applicable to $R_{cc}$ to obtain $\{w_k\}_{k=1}^{K}$. 
The pair matching method is used to achieve a computationally efficient estimation. We did the estimation and compared with our novel algorithm.

3.2. Simulation

The simulation for DOA based on MUSIC, ESPRIT and our novel algorithm for 2D are simulated through MATLAB 13.

Pattern due to MUSIC

Figure 1: Rectangular pattern due to MUSIC

Figure 2: Polar pattern due to MUSIC
Radiation pattern based on ESPRIT algorithm

Figure 3: Rectangular pattern due to ESPRIT

Figure 4: Polar pattern due to ESPRIT
Patterns due to our novel algorithm

Figure 5: Rectangular pattern due to Novel Algorithm

Figure 6: Polar pattern due to Novel Algorithm
Figure 7: Actual freq. Vs Actual DOA

Figure 8: Estimated Freq. Vs Estimated DOA
4. OBSERVATION

We did simulation of DOA using MUSIC and our new algorithm and compare them. From the Polar plot it is observed that the minor lobe is substantially cancelled and the major lobe is more directional towards the desired angle.

The DOA estimation of both algorithms are presented in tabular form. There are 6 pair of angles are considered between $0 < \alpha_i < 90^\circ$.

<table>
<thead>
<tr>
<th>ANGLE in DEGREE</th>
<th>DOA BASED ON MUSIC</th>
<th>DOA BASED ON NOVEL ALGORITHM</th>
</tr>
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<tbody>
<tr>
<td>ELIVATION</td>
<td>AZIMUH</td>
<td></td>
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<tr>
<td>20</td>
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<td>25</td>
<td>60.0032</td>
</tr>
<tr>
<td>70</td>
<td>15</td>
<td>70.0125</td>
</tr>
</tbody>
</table>

At $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$, are 20 45;30 55;40 48;50 37;60 25;70 15.

The comparative DOA based on MUSIC and our proposed algorithm are presented in the above table.

5. CONCLUSIONS

For DOA estimation through MUSIC, it require a priori knowledge of spatial background noise and interferences. The performance increased if we increase the no of elements of array. In the case of ESPRIT is required minimum 2 sub arrays and very complex computation.

Our proposed algorithm is very simple & it neither required a priori knowledge of background noise nor it needs complex calculations. It consumes minimum time for calculation the DOA. A adequate hardware can be designed for use. The main lobe is more directional and side lobes are cancelled and more reduced w.r.t that of MUSIC and ESPRIT.

REFERENCES


