A FINITE HORIZON INVENTORY MODEL WITH LIFE TIME, POWER DEMAND PATTERN AND LOST SALES

Vipin Kumar & S. R. Singh

Abstract: In this paper, the authors attempted to develop a deterministic inventory model for deteriorating item under a very realistic and practical demand rate which depends upon power of time (Power demand pattern). Also in this study we consider that the item deteriorate after a fixed time period called life time and taking incremental holding cost. Effect of partial backlogging also taken in account. Sensitivity analyses of some parameter also have been discussed.

1. INTRODUCTION

Many inventory models in past were developed under the assumptions that the holding cost is constant for the entire inventory cycle. But this is particularly false in the storage of deteriorating and perishable items such as food products. The longer these food products are kept in storage, the more sophisticated the storage facilities and services needed and therefore, the higher the holding cost. The holding cost is assumed to be varying over time in only few inventory models. The effect of time, storage conditions, weather conditions etc. are those factors which affect the quantity and quality of stock stored in the warehouse. Various kinds of materials are stored in the warehouse. Each material has its own characteristics. Some of the materials are affected by environmental conditions, the method of storage or the time of storage. Even later on, as researchers started taking other factors responsible for the decay of inventory into account, they stuck to comparatively primitive situations. Mostly, a linear and time dependent rate of decay was considered, which assumed that with time, the rate of decay of any commodity will go on increasing. This is still a comparatively better assumption, since very often it is observed, that once anything starts to spoil, its rate of spoilage goes on increasing by the day. Although it is simple, but still, time dependent linear rate of deterioration is a bit closer to reality than a constant rate. Since, it is unjustified to assume that any item starts deteriorating as soon as it is produced, hence, the item has been allowed a definite lifetime during which there is no deterioration and the item is able to sustain its qualities. After that, deterioration sets in, and it has been made more realistic and practical by taking Weibull distribution function. The competitive nature of the market has been accounted for by taking partial, time dependent, backlogging into consideration. The model is developed for a finite planning horizon, keeping in mind the short lifespan of the customer’s preferences.
Controlling inventories of perishable items poses a significant challenge due to limited useful life of items. These items if not used before the expiry date would outdate and there would be an additional cost of outdating of perished items. To maximize total reward over a finite horizon, price promotions can be used to clear off the sale of items having less remaining useful life. In a price sensitive market price promotion can be a reasonable option to stimulate the demand. Problem considered here is to determine an optimal time to announce price promotion and optimal ordering quantity in each period for a perishable item over a finite horizon. It is assumed that after the fixed horizon the product has to be withdrawn from the market. This is a quite realistic assumption as due technological advancement or to be competitive in market, management may withdraw the old products and introduce new ones. Perishability refers to decrease in value or usability of product over time due to the inherent characteristics of product; whereas obsolescence refers to loss in value of product due external factors such as, technological innovations, new product introduction by competitor, etc.

The literature on perishable inventory to determine optimal ordering policies, considered different scenarios related to demand patterns, issuing policies, review periods of inventory, etc. Gupta and Vrat (1986) were amongst the first few researchers to deliberate the effects of stock dependent consumption rate on an EOQ model. In this study they established EOQ for two cases, one for an instantaneous replenishment and another for a finite rate of replenishment. Baker and Urban (1988) analyzed a continuous deterministic case of an inventory system in which the demand rate is a polynomial function of the inventory level. The algorithm using separable programming was employed to find the optimal solution. Mandal and Phaujdar (1989) wrote a note an inventory model with instantaneous stock replenishment and stock dependent consumption rate. Datta and Pal (1990) developed an inventory model with stock dependent demand until the stock level reached a particular point, after which the demand became constant. Giri et al., (1996) extended Datta and Pal by relaxing their restriction of zero inventories at the end of order cycle and including deterioration effects. Balkhi and Benkherouf (2004) analyzed a deteriorating stock dependent model for a finite horizon. Mahapatra and Maiti (2005) set forth a study on multi objective inventory models with stock and quality dependent demand. Roy and Chaudhuri (2006) studied a model with stock dependent demand under inflation and constant deterioration. Chung and Lin (2001) presented an optimal inventory replenishment models for deteriorating item taking account of time discounting. Papachristos and Skouri (2003) extended the Wee (1999) model with the demand rate is a convex decreasing function of the selling price and the backlogging rate is a time-dependent. Yang (2005) developed a comparison among various partial backlogging inventory lot size models for deteriorating items on the basis of maximum profit. Roy A., (2008) introduced a deterministic inventory model for deteriorating items with price dependent demand and time varying holding cost.
He also considered, deterioration rate is time proportional and demand rate is a function of selling price.

Giri et al., (1996) developed an EOQ model for deteriorating items with shortages, in which both the demand rate and the holding cost are continuous functions of time. Shao et al., (2000) determined the optimum quality target for a manufacturing process where several grades of customer specifications may be sold. Since rejected goods could be stored and sold later to another customer, variable holding cost is considered in the model. In this model two type time dependent holding cost step functions are considered, retroactive holding cost increase and incremental holding cost increase. Most models that consider the deteriorating items are deteriorate from zero time. But this is not true in realistic, since each item deteriorate after a fixed time period called life time. There are few models are developed in which life time has been taken an important factor. Hwang and Hahn (2000) developed an optimal procurement policy for items with an inventory level dependent demand rate and fixed life time. Singh et al., (2005) presented an inventory model with life time, random rate of deterioration and partial backlogging. Balkhi and Benkherouf (2004) analyzed a deteriorating stock dependent model for a finite horizon. Mahapatra and Maiti (2005) set forth a study on multi objective inventory models with stock and quality dependent demand. Roy and Chaudhuri (2006) studied a model with stock dependent demand and constant deterioration. Alfares (2007) developed an inventory model wish stock level dependent demand rate and variable holding cost.

In this paper, an inventory model is developed for deteriorating item considering a power law form of the time dependence of demand. Also in this study we consider that the item deteriorate after a fixed time period called life time and taking incremental holding cost. Effect of partial backlogging and storage are also taken in account. A comprehensive sensitivity analysis has also been done for some parameters. Cost minimization technique is used to get the approximate expressions for total cost and other parameters. The model in developed and analyzed with the use of following assumption and notation.

2. ASSUMPTION AND NOTATION
The following assumption and notation are made in developing the present mathematical models of the inventory system:

$I(t)$ is the inventory level at any time $t$, $t \leq 0$

$I'(t)$ is the first derivative with respect to time

$\Theta(t) = \Theta(t)$ is variable rate of deterioration

$t_1$ is the time at which shortage starts and $T$ is the length of replenishment cycle $0 \leq t_1 \leq T$
$S$ is the initial inventory after fulfilling the backorders.

$C'$ denote the set up cost for each replenishment.

c3 and c1 denote the shortage cost for backlogged item and the unit cost of lost sales respectively and c0 be the deterioration cost per unit item per unit time

$\mu_0$ is the life time of items and deterioration of the items is considered only after the life time of the items

$D(t) = \frac{\lambda^{1-\alpha}}{\alpha^{\alpha}}$ is the demand rate at any time and $D(t) = \frac{\lambda^{1-\alpha}}{\alpha^{\alpha} + \alpha(t-\mu_0)}$

Shortages are allowed and backlogging rate is $D(t) = \frac{\lambda^{1-\alpha}}{\alpha^{\alpha} + \alpha(t-\mu_0)}$

When inventory is in shortage. The backlogging parameter $\alpha$ is positive constant and $0 \leq \alpha << 1$.

3. FORMULATION AND SOLUTION OF THE MODEL:

Let us assume after fulfilling backorders we get an amount $S (S \leq 0)$ as initial inventory. During the period $[0, \mu_0]$, the inventory level decrease due to the market demand only. After life time, i.e. during the period $[\mu_0, t_1]$ the inventory level decreases due to market demand and deteriorating of items and fall to zero at time $t_1$. The period $[t_1, T]$ is the period of the shortage which is partially backlogged. Also we assume that the holding cost increase time to time.
The differential equation governing the inventory level \( I(t) \) at any time \( t \) during the cycle \([0, T]\) are

\[
\frac{dI(t)}{dt} = -D(t) \quad 0 \leq t \leq \mu_0 \quad (1)
\]

\[
\frac{dI(t)}{dt} + \Theta(t)I(t) = -D(t) \quad \mu_0 \leq t \leq t_1 \quad (2)
\]

\[
\frac{dI(t)}{dt} = -\frac{D(t)}{1 + \alpha(T - t)} \quad t_1 \leq t \leq T \quad (3)
\]

The boundary condition are \( I(t) = S \) at \( t = 0 \) and \( I(t) = 0 \) at \( t = 0 \).

Solution of equation (1), (2) and (3) with the help of suitable boundary condition are given by

\[
I(t) = S - \frac{dt^{\gamma}}{T^{\gamma/2}} \quad 0 \leq t \leq \mu_0 \quad (4)
\]

\[
I(t) = \frac{d}{T^{\gamma/2}} \left[ \left(\frac{1 - \theta t^2}{2}\right) \left(t_1^{\gamma/2} - t^{\gamma/2}\right) + \frac{\theta}{2(2n-1)} \left(t_1^{2n+1/2} - t^{2n+1/2}\right)\right] \quad 0 \leq t \leq \mu_0 \quad (5)
\]

And

\[
I(t) = \frac{d}{T^{\gamma/2}} \left[ (1 - \alpha T) \left(t_1^{\gamma/2} - t^{\gamma/2}\right) + \frac{\alpha}{(n+1)} \left(t_1^{n+1/2} - t^{n+1/2}\right)\right] \quad t_1 \leq t \leq T \quad (6)
\]

By applying the boundary condition, \( S \) can be obtained as

\[
S = \frac{d\mu_0^{\gamma/2}}{T^{\gamma/2}} + \frac{d}{T^{\gamma/2}} \left[ \left(t_1^{\gamma/2} - \mu^{\gamma/2}\right) + \frac{\theta}{2(2n+1)} \left(t_1^{2n+1/2} - \mu_0^{2n+1/2}\right) - \frac{\theta\mu_0^2}{2} \left(t_1^{\gamma/2} - \mu^{\gamma/2}\right)\right] \quad (7)
\]

\[
\text{Total holding cost } C_H \text{ can be obtained as}
\]

\[
C_H = \int_0^{\mu_2} I(t) \, dt + \sum_{\mu_0}^{\mu_1} I(t) \, dt + \sum_{\mu_1}^{\mu_2} I(t) \, dt + \ldots + \sum_{\mu_{n-1}}^{\mu_n} I(t) \, dt
\]

\[
= \int_0^{\mu_2} I(t) \, dt + \sum_{j=1}^{m} c_j \int_{\mu_{j-1}}^{\mu_j} I(t) \, dt
\]
Total deteriorated cost $C_D$ during the period $[0, T]$ is given by

$$C_D = c_D \int_0^T \theta(t) I(t) \, dt$$

$$= c_D \frac{d\theta}{T^n} \int_0^t \left( t_1^{1/n} - t^{1/n} \right) \, dt$$

$$= c_D \frac{d\theta}{T^n} \left[ \frac{1}{2} \left( t_1^{2/n} - t^{2/n} \right) - \frac{n}{2n+1} \left( t_1^{2+n/1} - t^{2+n/1} \right) \right]$$

(Neglecting the power of $\theta$ higher than one)

Total shortage deteriorated cost $C_S$ during the period $[0, T]$ is given by

$$C_S = -c_S \int_0^T \theta(t) I(t) \, dt$$

$$= -c_S \frac{d\theta}{T^n} \left[ \frac{n}{n+1} T^{n/1} + \frac{1}{n+1} t_1^{n+1/1} - (\alpha T - 1) t_1^{1/T} \right]$$
Total lost sales cost $C_L$ during the period $[0, T]$ is given by

$$C_L = c_i \int_{t_i}^{T} \left\{ 1 - \frac{1}{1 + \alpha(t - T)} \right\} D(t) \, dt$$

$$= c_i \frac{d\alpha}{T^{\nu}} \left[ \frac{n}{n+1} T^{n/n} + \frac{1}{n+1} t_i^{n/n} - t_i^{n/n} T \right].$$ \hspace{1cm} (11)$$

Now the average cost of the system per unit time is given by

$$K = \frac{1}{T} \left[ C^i + C_H + C_D + C_S + D_H \right]$$ \hspace{1cm} (12)$$

The necessary condition for the total cost per unit time to be minimize are

$$\frac{\partial K}{\partial t_i} = 0$$ \hspace{1cm} (13)$$

And

$$\frac{\partial K}{\partial T} = 0$$ \hspace{1cm} (14)$$

provided the following condition

$$\frac{\partial^2 K}{\partial t_i} > 0, \quad \frac{\partial^2 K}{\partial T} > 0, \quad \text{and} \quad \left( \frac{\partial^2 K}{\partial t_i} \right) \left( \frac{\partial^2 K}{\partial T} \right) - \left( \frac{\partial^2 K}{\partial t_i \partial T} \right) > 0$$ \hspace{1cm} (15)$$

The equation (13) and (14) are gives

$$\left[ \frac{c_0 \mu_0}{2} \left( 2 + \theta(t_i^2 - \mu_0^2) \right) + \frac{1}{6} \sum_{j=1}^{\infty} \theta c_j \left( (t_i - \mu_{j-1}) \right) + 2 \theta \mu_i^3 + \theta \mu_{j-1}^3 - 3 \mu_{j-1} t' \right]$$

$$+ \frac{\theta c_d}{2} (t_i^2 - \mu_0^2) + (t_i - T)[c_s \left( 1 + \alpha(t_i - T) \right) + c_s \alpha] = 0.$$
And

\[
C^1 T^{\gamma/n} + \frac{n+1}{n} d c_0 \left[ n \left( T^{\gamma/n} + \frac{1-2\alpha T}{n+1} T^{\gamma/n} + (\alpha T - 1) T^{\gamma/n} \right) - \frac{2\alpha n}{(n+1)(2n+1)} T^{2\gamma/n} + \frac{\alpha}{2n+1} T^{2\gamma/n} \right] + d T \left[ (c_s + \alpha c_1) T^{\gamma/n} + (c_s (2\alpha T - 1) - \alpha c_1) T^{\gamma/n} - \frac{2\alpha c_s}{n+1} T^{2\gamma/n} \right] = 0.
\]

4. NUMERICAL ILLUSTRATION

To illustrate the model numerically the following parameter values are considered

\[
d = 50 \quad C^1 = \text{Rs. 200 per order}
\]
\[
n = 2 \quad c_0 = \text{Rs. 3 per unit per order}
\]
\[
\alpha = 0.1 \text{ units} \quad c_r = \text{Rs.} \ (c_0 + r) \text{ per unit per year}
\]
\[
\theta = 0.02 \text{ unit} \quad c_d = \text{Rs.} \ 10 \text{ per year}
\]
\[
\mu_0 = 0.4 \text{ year} \quad c_s = \text{Rs.} \ 12 \text{ per unit per order}
\]
\[
T = 1 \text{ year} \quad c_1 = \text{Rs.} \ 4 \text{ per unit}
\]

Taking \( m = 1 \), then for the minimization of total average cost, optimal solution is

\[
t_1 = 0.805092
\]
\[
S = 44.89025
\]
\[
K = 245.26614
\]
Table 1
Optimal Solution for Various Values of Deterioration Rate ($q$)

<table>
<thead>
<tr>
<th>Parameter value</th>
<th>% Change</th>
<th>$t_1^*$</th>
<th>$S^*$</th>
<th>$K^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.012</td>
<td>-40</td>
<td>0.80697</td>
<td>44.93202</td>
<td>245.10144</td>
</tr>
<tr>
<td>0.016</td>
<td>-20</td>
<td>0.80625</td>
<td>44.91731</td>
<td>245.15496</td>
</tr>
<tr>
<td>0.020</td>
<td>0</td>
<td>0.80509</td>
<td>44.89025</td>
<td>245.20198</td>
</tr>
<tr>
<td>0.024</td>
<td>+20</td>
<td>0.80416</td>
<td>44.86950</td>
<td>245.25202</td>
</tr>
<tr>
<td>0.028</td>
<td>+40</td>
<td>0.80288</td>
<td>44.83879</td>
<td>245.29693</td>
</tr>
</tbody>
</table>

The following points are noted from above table.

(i) The total average cost $K^*$ increases as the deterioration rate ($\theta$) increases.
(ii) Inventory period $t_1^*$ decreases as deterioration rate increases.
(iii) The initial inventory also decreases as deterioration rate increases.

Table 2
Optimal Solution for Various Values of Life Time ($\theta$)

<table>
<thead>
<tr>
<th>Parameter value</th>
<th>% Change</th>
<th>$t_1^*$</th>
<th>$S^*$</th>
<th>$K^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.24</td>
<td>-40</td>
<td>0.78140</td>
<td>44.23818</td>
<td>247.75075</td>
</tr>
<tr>
<td>0.32</td>
<td>-20</td>
<td>0.79263</td>
<td>44.54837</td>
<td>246.31703</td>
</tr>
<tr>
<td>0.40</td>
<td>0</td>
<td>0.80509</td>
<td>44.89025</td>
<td>245.20198</td>
</tr>
<tr>
<td>0.48</td>
<td>+20</td>
<td>0.81808</td>
<td>44.24396</td>
<td>244.35958</td>
</tr>
<tr>
<td>0.56</td>
<td>+40</td>
<td>0.83163</td>
<td>44.61078</td>
<td>243.77840</td>
</tr>
</tbody>
</table>

The following points are noted from above table.

(i) The total average cost $K^*$ decreases as life time of the items increase.
(ii) $t_1^*$ increase as life time of the items increase.
(iii) The initial inventory $S^*$ increase as life time of the items increase.

Table 3
Optimal Solution for Various Values of Backlogging Parameter ($\alpha$)

<table>
<thead>
<tr>
<th>Parameter value</th>
<th>% Change</th>
<th>$t_1^*$</th>
<th>$S^*$</th>
<th>$K^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.06</td>
<td>-40</td>
<td>0.80422</td>
<td>44.86591</td>
<td>245.14007</td>
</tr>
<tr>
<td>0.08</td>
<td>-20</td>
<td>0.80466</td>
<td>44.87807</td>
<td>245.16814</td>
</tr>
<tr>
<td>0.10</td>
<td>0</td>
<td>0.80509</td>
<td>44.89025</td>
<td>245.20198</td>
</tr>
<tr>
<td>0.12</td>
<td>+20</td>
<td>0.80553</td>
<td>44.90245</td>
<td>245.23284</td>
</tr>
<tr>
<td>0.14</td>
<td>+40</td>
<td>0.80596</td>
<td>44.91461</td>
<td>245.26364</td>
</tr>
</tbody>
</table>
The following points are noted from above table.

(i) The total average cost \( K^* \) decrease as backlogging parameter increase.
(ii) Inventory period \( t_1^* \) increase as backlogging parameter increase.
(iii) The initial inventory \( S^* \) increase as backlogging parameter increase.

5. CONCLUSIONS

In this paper, we have attempted to develop a deterioration inventory model with a very realistic and practical demand rate which depends upon power function of time. The rate of deterioration is variable. Also in this paper we consider that the item deteriorate after a fixed time period called life time and taking incremental holding cost. Allowable shortage is considered. Effect of partial backlogging also taken in account.

Numerical examples and sensitivity analysis has also been done for some parameter which show that the optimal production strategy may vary with changes in system parameters. Thus the authors hope that the present paper has a wide scope in realistic situations and can be applied in many real business situation.

REFERENCES

A Finite Horizon Inventory Model with Life Time, Power Demand Pattern and Lost Sales


Vipin Kumar  
Dept. of Mathematics, BKBIEET, Pilani (Raj.), India  
E-mail: vipinbkbiet@yahoo.co.in

S. R. Singh  
Dept. of Mathematics, D.N.College Meerut, India  
E-mail: shivrajpundir@yahoo.com