INVESTIGATION OF CAPACITY BOUNDS FOR MIMO SYSTEM FOR WIRELESS COMMUNICATIONS

T. Nageswara Rao¹, V. Srinivasa Rao² & M. Basha³

¹Department of Electronics & Communication Engineering, Anurag Engineering College, JNT University, Andhrapradesh, India, (E-mail: nagu.iitkgp17@gmail.com)
²Department of Electronics & Communication Engineering, Anurag Engineering College, JNT University, Andhrapradesh, India, (E-mail: vsrao.anurag@gmail.com)
³Department of Electronics & Communication Engineering, Anurag Engineering College, JNT University, Andhrapradesh, India, (E-mail: basha.abc@gmail.com)

Abstract: MIMO systems are used in wireless communications for enhancement of capacity. The number of transmit and receive antennas effect the tradeoff between performance gain and that of multiplexing gain. In this paper we are considering the study of Rayleigh fading affecting the capacity of MIMO channel by mat lab simulation. The Singular Value Decomposition (SVD) has been used to explore further insight into the property of the received signal and using which the water filling algorithm for the problem of maximizing the capacity of MIMO channel, composed of a set of parallel sub channels is implemented. The aim of the paper is to carry out investigations on MIMO system capacity and to observe the performance of the MIMO system.

Keywords: MIMO, SVD, MAC, SISO, Channel Capacity.

1. INTRODUCTION

The multiple input multiple output (MIMO) scheme deploys multiple number of antennas at the transmitter as well as at the receiver say $t$ and $r$ respectively. The data stream from a single user is demultiplexed into $t$ lower rate data streams and each stream is fed into one of the $t$ transmitting antennas all of which radiate at the same time in the same frequency band. By sharing the same frequency band the spectral efficiency is improved by min $\{r, t\}$ times. The receiver is assumed to have ideal channel estimates and so it can separate and decode the symbols transmitted from each antenna [1][2]. All the detected lower rate symbols from different streams are then multiplexed together to get the original high rate bit stream. The ability to separate out the symbols is due to the fact that in a scattering environment which is common in wireless mobile channels, the signals received at each receiving antenna from each transmitting antenna appear to be uncorrelated. Shannon defined the capacity of a channel as the maximum data rate at which data transmitted from a transmitter, when passed through the channel, can be received at some receiver with negligible chance of error. If the data source and received data are viewed as random variables, then the channel capacity refers to the maximum mutual information between them. The capacity $C$ is $C = \max I(X; Y)$. For a band limited channel with noise being Gaussian and white, Shannon derived the normalized capacity (capacity per unit bandwidth) to be $C = \log_2 (1 + \rho)$ bps/Hz. $\rho$ is the SNR at the receiver.

1.1. Multiuser MIMO System

Multi-user-MIMO is a space division multiple access, which allows a terminal to transmit or receive signal to or from multiple users say $k$ in the same band simultaneously. Multi user MIMO can be generalized into two categories, MIMO broadcast and MIMO multiple access channels for downlink and uplink situations respectively.

MIMO broadcast represents a MIMO downlink case and MIMO MAC represents a MIMO uplink case in which multiple senders transmit to single receiver at the base station. Each user in MU-MIMO has his own channel matrix. MIMO MAC systems outperform point-to-point MIMO systems especially when the number of receiver antennas at the receiving base station is larger than the number of
transmits antennas at each user [3] [7]. In this project we consider the study of MIMO MAC for the transmitter power optimization of all the \( k \) users in a MU-MIMO scenario, to maximize the sum rate or the sum capacity of the overall MIMO Multiple Access Channel (MIMO-MAC). The sum capacity of the multi-user system is a measure for the capacity efficiency of the whole single-cell system including all the \( k \) users.

Figure 1: MIMO System.

2. THEORY OF MIMO SYSTEM CAPACITY ANALYSIS

2.1. MIMO System Model

Consider a narrow-band single user MIMO system with \( t \) transmit and \( r \) receive antennas. The antennas are assumed to be Omni directional, which means that the antennas transmit and receive equally well in all directions.

The linear link model between the transmit and receive antennas can be represented in the vector notation as \( y = H x + n \). Where \( x \) is the \((t \times 1)\) transmit vector, \( y \) is the \((r \times 1)\) receive vector, \( H \) is the \((r \times t)\) channel matrix and \( n \) is the \((r \times 1)\) additive white Gaussian noise vector.

The channel is assumed to be flat (narrow bandwidth) and slow fading so it does not change during a burst of transmission. Each entry of \( H \), \( h_{ij} \) represents the path gain between the \( j \)th transmit antenna and \( i \)th receive antenna [4]. The \( h_{ij} \) is Rayleigh distributed and in a rich scattering environment the columns of \( H \) are assumed to be independent. Also, when the transmitter has no knowledge about the channel, it is optimum to use equal power distribution on each antenna and PT being the total signal power available at the transmitter, the MIMO channel capacity for equal power \( C = \log_2 \left| \text{det} \left( I_r + \left( \frac{\rho}{t} \right)^* HH^\dagger \right) \right| \) where \( \rho = PT/(\sigma)^2 \) is the average SNR at each receiving antenna and \( H^\dagger \) is the complex conjugate transpose of \( H \), \( I_r \) is the \((r \times r)\) identity matrix and \( (\sigma)^2 \) is the noise power on each receiving antenna. In order to investigate the characteristics of \( H \), we perform the Singular Value Decomposition on \( H \) to diagonalizable \( H \) and find the Eigen values.

There are Four Cases to observe (1) Study of Water filling algorithm. (2) Implementation of water filling algorithm for capacity enhancement of single user MIMO channel (3) Study of bit error rate behavior when water filling algorithm is used for unequal power distribution among the sub channels (4) Study of extension of single user MIMO to Multi User MIMO (MU-MIMO) for sum rate capacity maximization of all \( K \) users in a MU-MIMO system for the MIMO-MAC.

3. WATER FILLING CAPACITY OF MIMO

Channel when the transmitter has perfect knowledge of the channel, Singular Value Decomposition (SVD) is performed on the channel matrix and the water filling method is used to optimize the transmitted signal power. Singular Value decomposition is used to decompose the MIMO channel matrix into parallel SISO sub channels with unequal channel gains. The principle of the water filling theorem sees the division of total power available at the transmitter in such a way that a greater portion goes to the sub channels with higher gain and less or even none to the channels with small gains. The sub channels with lower gain i.e. those with higher noise for which no power is allocated at all refer to those sub channels which are not used for transmitting any signal during the transmission [5][6]. The objective of this algorithm is to allocate power across the sub channels unequally so as to maximize the total capacity of the overall MIMO channel. This power allocation is subject to the constraint that the total sum of the power poured into all sub channels is equal to PT, the total power available at the transmitter.

The relative channel strengths and the amount of power to allocate to each channel is determined by knowledge of the channel matrix, \( H \). We use the singular value decomposition of \( H \) to obtain \( H(rXt) = U D V^\dagger \); Where \( U U^\dagger = I_r \) and \( V V^\dagger = I_t \) and \( D = \text{diagonal} \{ \lambda_1, \lambda_2, \lambda_3, ..., \lambda_n \} \) with \( \lambda_i \) as the positive square root of \( \lambda_i \)th eigen value and \( i = 1 \) to \( n \) non zero \( \lambda \) values and \( n = \min \{ r, t \} \).
3.1. Summary of Steps Involved in the Water Filling Power Allocation to the MIMO Sub Channels

1. The first step is to determine the water filling parameter or threshold, \( \mu \), which is also called the water level. Note that the \( \mu \) is just a mathematical parameter used to determine the power allocated to each of the eigen channels.

2. After determining \( \mu \), the inverse of eigen values of the matrix \( H \) are compared with the threshold \( \mu \).

3. Now if \( \lambda_i^2 \geq \mu \) then, the gain of the \( i^{th} \) eigen channel is too small and this eigen channel will not be considered for communication.

4. Assume the case of a square dimension of MIMO channel, i.e. \( r = t \), and also \( \lambda_i \geq \lambda_2 = \lambda_3 \geq \ldots \lambda_m \). And also consider that \( m \) eigen values have survived after the above described procedure in step 3.

5. Once the total available power, \( P_T \), and the gains of the parallel sub channels are known, the optimum power allocated to the \( i^{th} \) sub channel is

\[
p_i = \left[ \mu - \frac{1}{\lambda_i^2} \right]
\]

And the power allocated to each of these eigen channels, \( p_i \), is determined by the water filling rule such that the following equations are satisfied.

\[
\frac{1}{\lambda_i^2} + p_1 = \frac{1}{\lambda_2^2} + p_2 = \ldots = \frac{1}{\lambda_m^2} + p_m = \mu
\]

for \( i = 1, 2, \ldots, m \). (2)

and

\[
p_i = p_1 + p_2 + \ldots + p_m
\]

(3)

If this quantity \( p_i = \left[ \mu - \frac{1}{\lambda_i^2} \right] \) is positive then the power \( P_i \) is allocated to the \( i^{th} \) sub channel otherwise, the sub channel is left unused. The water filling parameter ‘\( \mu \)’ is determined by the total power \( P_T \) as shown below.

\[
\frac{1}{\lambda_i^2} + P_1 = \frac{1}{\lambda_2^2} + P_2 = \ldots = \frac{1}{\lambda_m^2} + P_m = \mu
\]

(4)

\[
P_T = p_1 + p_2 + \ldots + p_m
\]

(5)

\[
m_\mu = \sum_{i=1}^{m} \frac{1}{\lambda_i^2} + \sum_{k=1}^{m} p_k = \sum_{i=1}^{m} \frac{1}{\lambda_i^2} + P_T
\]

(6)

\[
m = \frac{1}{m} \left[ \sum_{i=1}^{m} \frac{1}{\lambda_i^2} + P_T \right]
\]

(7)

Where \( m \) is the number of sub channels that have survived after the cut off elimination process.

The weakness of the sub channel is basically determined by \( \mu \). The larger the value of \( \mu \), the more the number of sub channels that are kept for using for transmission. Also \( \mu \) is itself dependent on the total power, \( P_T \), available at the transmitter and the Eigen values of the MIMO channel matrix \( H \). Now the capacity of MIMO channel with water filling can be expressed as

\[
C = \sum_{i=1}^{m} \log_2 \left[ 1 + \frac{P_i}{\sigma^2} \right] \text{bps/Hz}
\]

(8)

4. RESULTS AND DISCUSSION

4.1. Matlab Simulation

The capacity of the MIMO channel has been simulated for various number of transmitter and receiver antennas using the water-filling algorithm for allocation of optimum power to the parallel sub channels, represented by the diagonal elements of the diagonal matrix which was obtained by performing the Singular Value Decomposition of the MIMO channel matrix. For each case of \( (tXr) \) 10,000 MIMO channel matrices have been generated and the mean capacity of the MIMO channel is plotted against SNR in db. The elements of the MIMO channel matrix generated in the mat lab simulation are Rayleigh distributed and each element in the matrix represents the gain that exists between a pair of Transmitting and Receiving antennas.

The Channel State Information (CSI) is assumed to be known at the transmitter and the generated channel matrix “\( H \)” is directly used to perform the Singular Value Decomposition to determine the parallel sub channel gains in the decomposed equivalent MIMO channel. The capacity of MIMO systems as a function of SNR for different \( (r, t) \) combinations is plotted in figure 2, using the water filling algorithm for allocating the transmitting power optimally over the various sub channels of the MIMO channel. From the capacity curves it can be seen that at higher SNR for a \( (1X1) \) system the increase in capacity is roughly 1 bit for a 3 db increase in SNR. Even for a \( (4X4) \) system in the MIMO the increase in mean capacity is about 4 bits for a 3 db increase in SNR. The figure 2 also shows the comparison of capacities of MIMO channel with and without using the water filling algorithm for the various cases of \( (rXt) \) MIMO pairs. The result obtained shows that there is an improvement in capacity of MIMO channel when the water filling solution to achieve capacity maximization is used.
to allocate different powers to the sub channels. And the enhancement of capacity is seen for lower levels of SNR.

The capacity of MIMO channel described by the equation 1 is instantaneous for a given realization of $H$ and since in Rayleigh fading the channel matrix changes randomly, the capacity also changes randomly. Therefore to incorporate this sense of randomness, a better way to express the capacity of MIMO channel is as follows. The figure 4, shows the curve of probability (capacity $>C_o$) for a SNR of 8 db and for various pairs of $(rXt)$ mimo antennas. The interpretation of this graph is that the Y axis gives the value of probability that the capacity values obtained from using various $(rXt)$ pairs is greater than a particular value $C_o$ on the X axis. The result obtained from simulation in the figure 4, clearly shows that for a SNR of 8 db, and for (8X8) of $(rXt)$ pair, for 90% of the times, the capacities extracted from the random MIMO channel are greater than 45 bps/HZ.

Another simulated graph for comparing the probabilities for two $(rXt)$ pairs, (1X1) and (4x4) for various different SNR values was obtained as shown in the figure 5.
5. CONCLUSION

This paper mainly gives investigations on capacity of MIMO channel. Some of the results obtained from Mat lab simulations are summarized in this paper and each graph is interpreted. The plot of capacity versus SNR in figure 1. Shows that the capacity of the MIMO channel increases as the number of antennas used at both the transmitter and the receiver, increases. 10,000 versions of random Rayleigh distributed MIMO channel matrices were generated for each case of (rXt) to get the capacity values and then the mean of the capacity was plotted against SNR in db. The water filling algorithm was implemented in this work to carry out simulations on the MIMO channel capacity. The simulated result of figure 2 shows that at higher SNR s the value of capacity for the (1X1) system increases by about 1 bit for a 3db increase in SNR. And also for the case of (4X4) of (rXt) pair the mean capacity increases by about 4 bits per 3 db increase in SNR. Hence it can be concluded that the capacity of a MIMO system increases linearly with the increase in number of antennas. The graph of figure 2 also shows an enhancement of capacity when water filling algorithm was used to allocate unequal powers to the different sub channels. The graph in figure 3. Shows that the bit error rate decreases when the number of antennas at both the transmitter and the receiver are increased. From this result we can conclude the fact that MIMO system offers a diversity gain and hence supports a better quality of communication.

REFERENCES


