Peristaltic Transport of a Jeffrey Fluid Through a Porous Medium in an Inclined Tube Under the Effect of a Magnetic Field

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ABSTRACT

We studied the peristaltic motion of MHD Jeffrey fluid through porous medium in an inclined tube under the assumption of long wavelength. Closed form solutions for velocity and pressure gradient are obtained. The effects of various emerging parameters on the pressure gradient, pumping characteristics and frictional force are discussed through graphs in detail.

Keywords: Darcy number, Froude number, Hartmann number, Inclination angle, Jeffrey fluid, Peristaltic transport.

1. INTRODUCTION

Peristaltic transport occurs in a wide variety of physiological and engineering applications such as urine transport in the ureter, motion of spermatozoa in the cervical canal, the movement of chyme in the gastrointestinal tract, swallowing of food through oesophagus, the vasomotion of small blood vessels, in roller and finger pumps and many others. After the seminal work of Latham [12], several researchers have studied the phenomenon of peristaltic transport under various assumptions. It is noticed from the available literature that much has been reported on the peristaltic transport of hydrodynamic viscous and non-Newtonian fluids. The good number of recent investigations (Siddiqui and Schwarz, [13]; Siddiqui and Schwarz, [14]; Hayat et al., [6]; Hayat et al., [7]; Subba Reddy et al., [19]) on the peristalsis of non-Newtonian fluids has been presented with various perspectives, in channels or tubes.

Magnetohydrodynamics (MHD) is the science which deals with the motion of a highly conducting fluid in the presence of a magnetic field. The motion of the conducting fluid across the magnetic field generates electric currents which change the magnetic field, and the action of the magnetic field on these currents gives rise to mechanical forces which modify the flow of the fluid (Ferraroand Plumpton, [5]). The MHD flow of a fluid in a channel with elastic, rhythmically contracting walls (peristaltic flow) is of interest in connection with certain problems of the movement of conductive physiological fluids, e.g., the blood and blood pump machines, and with the need for theoretical research on the operation of a peristaltic MHD compressor. The effect of moving magnetic field on blood flow was studied by Stud et al., [18], and they observed that the effect of suitable moving magnetic field accelerates the speed of blood. Srivastava and Agrawal [17] considered the blood as an electrically conducting fluid and constitutes a suspension of red cell in plasma. Agrawal and Anwaruddin [3] studied the effect of magnetic field on blood flow by taking a simple mathematical model for blood through an equally branched channel with flexible walls executing...
peristaltic waves using long wavelength approximation method and observed, for the flow blood in arteries with arterial disease like arterial stenosis or arteriosclerosis, that the influence of magnetic field may be utilized as a blood pump in carrying out cardiac operations. Abd El Hakeem et al., [1] have studied the effects of a uniform magnetic field on trapping at centerline and at the channel wall for Carreau fluid through uniform channel. Also, Hayat et al., [8] discussed the peristaltic flow of a MHD fourth grade fluid in a channel under the consideration of long wavelength and low Reynolds number, and they considered the fluid as electrically conducting in the presence of a uniform transverse magnetic field. Eldabe et al., [4] have studied the Peristaltic transport of a MHD biviscosity fluid in a non-uniform tube. Peristaltic flow of Jeffrey fluid in a uniform tube under the effect of magnetic field was studied by Hayat and Ali [9]. Hayat et al., [10] have investigated the effect of magnetic filed on the peristaltic transport of Jeffrey fluid trough a porous medium in a channel with compliant walls. Affifi and Gad [2] have studied the interaction of peristaltic flow with pulsatile magneto fluid through a porous medium. Srinivas and Kothandapani [16] have studied the influence of heat and mass transfer on the MHD peristaltic flow through a porous medium with compliant walls. Mekheimer [13] have analyzed the peristaltic flow of a viscous fluid in an inclined channel under the effect of a magnetic field. Hayat et al., [11] have studied the effect of inclined magnetic field on peristaltic transport of fourth grade fluid in an inclined asymmetric channel.

However, the interaction of magnetic field with peristaltic flow of a Jeffrey fluid through a porous medium in an inclined tube has received little attention. Hence, an attempt is made to study the peristaltic motion of MHD Jeffrey fluid through porous medium in an inclined tube under the assumption of long wavelength. Closed form solutions for velocity and pressure gradient are obtained. The effects of various emerging parameters on the pressure gradient, pumping characteristics and frictional force are discussed through graphs in detail.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

We consider the axisymmetric flow of a Jeffrey fluid in an inclined uniform circular tube with inclination induced by a sinusoidal peristaltic wave of small amplitude travelling down its wall. Fig. 1 shows the physical model of the problem. The geometry of wall surface is therefore described as

\[
H(Z, t) = a + b \cos \frac{2\pi}{\lambda} (Z - ct).
\]

where \(a\) is the radius of the undisturbed tube, \(b\) is the amplitude of the peristaltic wave, \(\lambda\) is the wavelength, \(c\) is the propagation speed, and \(t\) is the time. \(R\) and \(Z\) are the cylindrical coordinate with measured along the axis of the tube and is in the radial direction.

![Figure 1: Physical Model](image-url)
Let \((U, W)\) be the velocity components in fixed frame of reference \((R, Z)\). We further assume that wall is extensible and fluid is electrically conducting. A uniform magnetic field \(B_0\) is applied in the transverse direction to the flow. The magnetic Reynolds number is taken small so that induced magnetic field is neglected.

The equations governing the flow in a fixed frame are given by

\[
\rho \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial R} + W \frac{\partial}{\partial Z} \right) U = -\frac{\partial p}{\partial R} + \frac{1}{R} \frac{\partial}{\partial R} (RS_{RR}) + \frac{\partial}{\partial Z} (S_{RZ}) - \frac{S_{\theta\theta}}{R} - \frac{\mu}{k} U - \rho g \cos \alpha \tag{2.3}
\]

\[
\rho \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial R} + W \frac{\partial}{\partial Z} \right) W = -\frac{\partial p}{\partial Z} + \frac{1}{R} \frac{\partial}{\partial R} (RS_{RR}) + \frac{\partial}{\partial Z} (S_{ZZ}) - \left( \frac{\mu}{k} + \sigma B_0^2 \right) W + \rho g \sin \alpha \tag{2.4}
\]

\[
\frac{\partial U}{\partial R} + \frac{U}{R} + \frac{\partial W}{\partial Z} = 0 \tag{2.5}
\]

where \(\rho\) is the density, \(\mu\) is the dynamic viscosity, \(g\) is the acceleration due to gravity, \(\sigma\) is the electrical conductivity of the fluid, \(U\) and \(W\) are the velocity component in \(R\) and \(Z\) directions, respectively and \(P\) is the pressure.

The constitutive equation for an incompressible Jeffrey fluid is

\[
S = \frac{\mu}{1 + \lambda_1} (\dot{\gamma} + \lambda_2 \dot{\gamma}) \tag{2.6}
\]

where \(S\) extra stress tensor respectively, \(I\) is the identity tensor, \(\lambda_1\) is the ratio of relaxation to retardation times, \(\lambda_2\) is the retardation time, is the dynamic viscosity, is the shear rate and dots over the quantities indicate differentiation with respect to time.

In the fixed frame of reference \((R, Z)\) the flow is unsteady. However, in a coordinate frame moving with the wave speed \(c\) (wave frame) \((r, z)\) the boundary shape is stationary. The transformation from fixed frame to wave frame is given below as

\[
r = R, \quad z = Z - ct, \quad u(r, z) = U, \quad w(r, z) = W - c \tag{2.7}
\]

\(u\) and \(w\) being the velocity components in the wave frame.

The boundary conditions in wave frame are

\[
\frac{\partial w}{\partial r} = 0 \quad \text{at} \quad r = 0 \tag{2.8}
\]

\[
w = -c \quad \text{at} \quad r = H \tag{2.9}
\]

The governing hydrodynamic equations are the equations of conservation of mass and momentum. The momentum equation here is modified to account for the interaction between magnetic field and fluid flow through the ponder motive force. The governing equations in the wave frame are given as follows:

\[
\rho \left( u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} (rS_{rr}) + \frac{\partial}{\partial z} (S_{rz}) - \frac{S_{\theta\theta}}{r} - \frac{\mu}{k} u - \rho g \cos \alpha \tag{2.10}
\]

\[
\rho \left( u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} (rS_{rr}) + \frac{\partial}{\partial z} (S_{zz}) - \left( \frac{\mu}{k} + \sigma B_0^2 \right) (w + c) + \rho g \sin \alpha \tag{2.11}
\]

\[
\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{2.12}
\]
where

$$S = \frac{\mu}{1 + \lambda_1} \left[ 1 + \lambda_2 \left( u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) \right] \gamma$$  \hspace{1cm} (2.13)$$

Introducing the following non-dimensional variables

$$\bar{r} = \frac{r}{a}, \quad \bar{z} = \frac{z}{\lambda}, \quad \bar{u} = \frac{\lambda u}{ac}, \quad \bar{p} = \frac{a^2 \rho}{c \lambda \mu}, \quad \bar{h} = \frac{H}{a}, \quad \bar{w} = \frac{w}{c}, \quad \bar{S} = \frac{a S}{\mu c}, \phi = \frac{a}{b}$$  \hspace{1cm} (2.14)$$

into Eqs. (2.10) and (2.11), we get (after dropping bars)

$$\text{Re} \delta \left( u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) u = - \frac{\partial p}{\partial r} + \frac{\delta}{r} \frac{\partial}{\partial r} \left( r S_{rr} \right) + \delta^2 \frac{\partial}{\partial z} \left( S_{rz} \right) - \frac{\delta^2}{r} \frac{S_{\theta \theta}}{Da} - \frac{\text{Re}}{\text{Fr}} u - \text{Re} \delta \cos \alpha$$  \hspace{1cm} (2.15)$$

$$\text{Re} \delta \left( u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) w = - \frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left( r S_{rr} \right) + \frac{\delta}{r} \frac{\partial}{\partial z} \left( S_{zz} \right) - N^2 (w + 1) + \frac{\text{Re}}{\text{Fr}} \sin \alpha$$  \hspace{1cm} (2.16)$$

The boundary conditions in dimensionless form are

$$\frac{\partial w}{\partial r} = 0 \quad \text{at} \quad r = 0$$  \hspace{1cm} (2.17)$$

$$w = -1 \quad \text{at} \quad r = h = 1 + \phi \cos 2\pi z$$  \hspace{1cm} (2.18)$$

where \(N^2 = M^2 + \frac{1}{Da}, M = \sqrt{\frac{\mu}{a^2} B_0},\) is the Hartman number, \(Da = \frac{k}{a^2}\) is the Darcy number, \(\delta = \frac{a}{\lambda}\) is the wave number, \(Fr = \frac{c}{ka}\) is Froude number, \(\text{Re} = \frac{k}{a^2}\) is the Reynolds number,

$$S_{rr} = \frac{2\delta}{1 + \lambda_1} \left[ 1 + \lambda_2 \frac{\partial}{a} \left( u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) \right] \frac{\partial u}{\partial r}, \quad S_{rz} = \frac{1}{1 + \lambda_1} \left[ \frac{\lambda_2 c \delta}{a} \left( u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) \right] \frac{\partial u}{\partial r} + \delta^2 \frac{\partial u}{\partial z},$$

$$S_{\theta \theta} = \frac{2\delta}{1 + \lambda_1} \left[ 1 + \lambda_2 \frac{\partial}{a} \left( u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) \right] \frac{u}{r}, \quad \text{and} \quad S_{zz} = \frac{2\delta}{1 + \lambda_1} \left[ 1 + \lambda_2 \frac{\partial}{a} \left( u \frac{\partial}{\partial r} + w \frac{\partial}{\partial z} \right) \right] \frac{\partial w}{\partial z}.$$ 

Under the assumption of long wavelength the Eq. (2.15) and (2.16) reduce to

$$0 = - \frac{\partial p}{\partial r},$$  \hspace{1cm} (2.19)$$

$$0 = - \frac{\partial p}{\partial z} + \frac{1}{r} u \frac{\partial}{\partial r} \left( \frac{r}{1 + \lambda_1} \frac{\partial w}{\partial r} \right) - N^2 (w + 1) + \frac{\text{Re}}{\text{Fr}} \sin \alpha.$$  \hspace{1cm} (2.20)$$

Here Eq. (2.19) indicates that \(p\) is independent of \(r\) and depends only upon \(z\). Therefore, Eq. (2.20), can be rewritten as

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} - (1 + \lambda_1) N^2 w = \left( \frac{dp}{dz} + N^2 - \frac{\text{Re}}{\text{Fr}} \sin \alpha \right) (1 + \lambda_1).$$  \hspace{1cm} (2.21)$$
The non-dimensional volume flow rate is given by

\[ q = \int_{0}^{h} wrdr. \]  

(2.22)

The instantaneous flux \( Q(Z, t) \) in the laboratory frame is

\[ Q(Z, t) = \int_{0}^{h} WRdR = \int_{0}^{h} (w + 1) rdr = q + \frac{h^2}{2}. \]  

(2.23)

The average flux over one period \((T = \lambda/c)\) of the peristaltic wave is

\[ \bar{Q} = \frac{1}{T} \int_{0}^{T} Qdt = \frac{1}{T} \int_{0}^{T} \left( q + \frac{h^2}{2} \right) dt = \int_{0}^{1} \left( q + \frac{h^2}{2} \right) dt = q + \frac{1}{2} \left( 1 + \frac{\phi^2}{2} \right). \]  

(2.25)

3. SOLUTION

The solution of Eq. (2.21) using the boundary conditions (2.17) & (2.18), we obtain

\[ w = \frac{1}{N^2} \left( \frac{dp}{dz} - \frac{Re}{Fr} \sin \alpha \right) \left[ \frac{I_0[N(\sqrt{1+\lambda_1})r]}{I_0[N(\sqrt{1+\lambda_1})h]} - 1 \right] - 1. \]  

(3.1)

Here \( I_0 \) is modified Bessel function of first kind of order zero.

The non-dimensional volume flow rate \( q \) is given by

\[ q = \int_{0}^{h} rwdr \]

\[ = \frac{1}{N^2} \left( \frac{dp}{dz} - \frac{Re}{Fr} \sin \alpha \right) \left\{ \frac{2h I_1[N(\sqrt{1+\lambda_1})h] - h^2 N(\sqrt{1+\lambda_1}) I_0[N(\sqrt{1+\lambda_1})h]}{2N(\sqrt{1+\lambda_1}) I_0[N(\sqrt{1+\lambda_1})h]} \right\} - \frac{h^2}{2}. \]  

(3.2)

Here \( I_1 \) is modified Bessel function of first kind of order one.

From Eq. (3.3), we have

\[ \frac{dp}{dz} = \frac{(2q + h^2) N^3(\sqrt{1+\lambda_1}) I_0[N(\sqrt{1+\lambda_1})h]}{2h I_1[N(\sqrt{1+\lambda_1})h] - h^2 N(\sqrt{1+\lambda_1}) I_0[N(\sqrt{1+\lambda_1})h]} + \frac{Re}{Fr} \sin \alpha. \]  

(3.3)

The pressure rise \((\Delta p)\) and frictional force \((F)\) on the tube wall per one wave length are defined as

\[ \Delta p = \int_{0}^{1} \left( \frac{dp}{dz} \right) dz, \]  

(3.4)

\[ F = \int_{0}^{h} \left( - \frac{dp}{dz} \right) dz. \]  

(3.5)

The above integrals numerically evaluated using the MATLAB software.
4. DISCUSSION OF THE RESULTS

Figure 2 shows the variation of axial pressure gradient $\frac{dp}{dz}$ with Darcy number $Da$ for $\phi = 0.6$, $M = 1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $Fr = 2$ and $\lambda_1 = 0.3$. It is found that, the pressure gradient oscillates with $z$. Also, it is observed that the axial pressure gradient $\frac{dp}{dz}$ decreases with increasing Darcy number $Da$.

![Image: Figure 2: The Variation of Axial Pressure Gradient $\frac{dp}{dz}$ with Darcy number $Da$ for $\phi = 0.6$, $M = 1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $Fr = 2$ and $\lambda_1 = 0.3$]

The variation of axial pressure gradient $\frac{dp}{dz}$ with Hartmann number $M$ for $\phi = 0.6$, $Da = 0.1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $Fr = 2$ and $\lambda_1 = 0.3$ is shown in Fig. 3. It is observed that, the axial pressure gradient $\frac{dp}{dz}$ increases with an increase in Hartmann number $M$.

![Image: Figure 3: The Variation of Axial Pressure Gradient $\frac{dp}{dz}$ with Hartmann Number $M$ for $\phi = 0.6$, $Da = 0.1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $Fr = 2$ and $\lambda_1 = 0.3$]

Figure 4 depicts the variation of axial pressure gradient $\frac{dp}{dz}$ with $\lambda_1$ for $\phi = 0.6$, $M = 1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $Fr = 2$ and $Da = 0.1$. It is noted that, the axial pressure gradient $\frac{dp}{dz}$ decreases with increasing $\lambda_1$.

![Image: Figure 4: The Variation of Axial Pressure Gradient $\frac{dp}{dz}$ with $\lambda_1$ for $\phi = 0.6$, $M = 1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $Fr = 2$ and $Da = 0.1$]
The variation of axial pressure gradient $\frac{dp}{dz}$ with inclination angle $\alpha$ for $\phi = 0.3$, $M = 1$, $\lambda_1 = 0.3$, $Re = 10$, $Fr = 2$ and $Da = 0.1$ is presented in Fig. 5. It is observed that, the axial pressure gradient $\frac{dp}{dz}$ increases with increasing inclination angle $\alpha$.

Figure 5: The Variation of Axial Pressure Gradient $\frac{dp}{dz}$ with Inclination Angle $\alpha$
for $\phi = 0.3$, $M = 1$, $\lambda_1 = 0.3$, $Re = 10$, $Fr = 2$ and $Da = 0.1$

Figure 6 illustrates the variation of axial pressure gradient $\frac{dp}{dz}$ with Froude number $Fr$ for $\phi = 0.3$, $M = 1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $\lambda_1 = 0.3$ and $Da = 0.1$. It is found that, the axial pressure gradient $\frac{dp}{dz}$ decreases with an increase in Froude number $Fr$.

Figure 6: The Variation of Axial Pressure Gradient $\frac{dp}{dz}$ with Froude Number $Fr$
for $\phi = 0.3$, $M = 1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $\lambda_1 = 0.3$ and $Da = 0.1$

The variation of axial pressure gradient $\frac{dp}{dz}$ with Reynolds number $Re$ for $\phi = 0.3$, $M = 1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $\lambda_1 = 0.3$, $Fr = 2$ and $Da = 0.1$ is depicted in Fig. 7. It is noticed that, the axial pressure gradient $\frac{dp}{dz}$ increases with increasing Reynolds number $Re$.

Figure 7: The Variation of Axial Pressure Gradient $\frac{dp}{dz}$ with Reynolds Number $Re$
for $\phi = 0.3$, $M = 1$, $\alpha = \frac{\pi}{4}$, $\lambda_1 = 0.3$, $Fr = 2$ and $Da = 0.1$
Figure 8 shows the variation of axial pressure gradient \( \frac{dp}{dz} \) with amplitude ratio \( \phi \) for \( \lambda_1 = 0.3, M = 1, \alpha = \frac{n}{4}, \) Re = 10, Fr = 2 and \( Da = 0.1. \) It is found that, the axial pressure gradient \( \frac{dp}{dz} \) increases with an increase in amplitude ratio \( \phi. \)

![Figure 8: The Variation of Axial Pressure Gradient \( \frac{dp}{dz} \) with Amplitude Ratio \( \phi \) for \( \phi = 0.3, M = 1, \alpha = \frac{n}{4}, \) Re = 10, Fr = 2 and \( Da = 0.1 \)](image)

Figure 9 depicts the variation of pressure rise \( \Delta p \) with \( \bar{Q} \) for different values of Darcy number \( Da \) with \( \phi = 0.6, M = 1, \alpha = \frac{n}{4}, \) Re = 10, Fr = 2 and \( \lambda_1 = 0.3. \) It is observed that, in the pumping region (\( \Delta p > 0 \)) the time averaged flux \( \bar{Q} \) decreases with increasing Darcy number \( Da, \) while it increases with increasing \( Da \) in both the free-pumping (\( \Delta p = 0 \)) and co-pumping (\( \Delta p < 0 \)) regions.

![Figure 9: The Variation of Pressure Rise \( \Delta p \) with \( \bar{Q} \) for Different Values of Darcy Number \( Da \) with \( \phi = 0.6, M = 1, \alpha = \frac{n}{4}, \) Re = 10, Fr = 2 and \( \lambda_1 = 0.3 \)](image)

The variation of pressure rise \( \Delta p \) with \( \bar{Q} \) for different values of Hartmann number \( M \) with \( \phi = 0.6, Da = 0.1, \alpha = \frac{n}{4}, \) Re = 10, Fr = 2 and \( \lambda_1 = 0.3 \) is shown in Fig. 10. It is found that, in the pumping region the time averaged flux \( \bar{Q} \) increases with increasing Hartmann number \( M, \) while it decreases with increasing \( M \) in both the free-pumping and co-pumping regions.

![Figure 10: The Variation of Pressure Rise \( \Delta p \) with \( \bar{Q} \) for Different Values of Hartmann number \( M \) with \( \phi = 0.6, Da = 0.1, \alpha = \frac{n}{4}, \) Re = 10, Fr = 2 and \( \lambda_1 = 0.3 \)](image)
Figure 11 shows the variation of pressure rise $\Delta p$ with $Q$ for different values of $\lambda_1$ with $\phi = 0.6$, $M = 1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $Fr = 2$ and $Da = 0.1$. It is noted that, the time averaged flux $\overline{Q}$ decreases with increasing $\lambda_1$ in both the pumping and free-pumping regions, while it increases with increasing $\lambda_1$ in the co-pumping region for appropriately chosen $\Delta p (\approx -4.512)$. Further it is observed that, the pumping is more for Newtonian fluid ($\lambda_1 \rightarrow 0$) than that of Jeffrey fluid ($0 < \lambda_1 < 1$).

The variation of pressure rise $\Delta p$ with $Q$ for different values of inclination angle $\alpha$ with $\phi = 0.6$, $M = 1$, $\lambda_1 = 0.3$, $Re = 10$, $Fr = 2$ and $Da = 0.1$ is presented in Fig. 12. It is found that, the time averaged flux $\overline{Q}$ increases with increasing inclination angle $\alpha$ in all the pumping, free-pumping and co-pumping regions.

Figure 12: The Variation of Pressure Rise $\Delta p$ with $Q$ for Different Values of Inclination Angle $\alpha$
with $\phi = 0.6$, $M = 0.1$, $\lambda_1 = 0.3$, $Re = 10$, $Fr = 2$ and $Da = 0.1$

Figure 13 shows the variation of pressure rise $\Delta p$ with $Q$ for different values of Froude number $Fr$ with $\phi = 0.6$, $M = 1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $\lambda_1 = 0.3$ and $Da = 0.1$. It is observed that, the time averaged flux $\overline{Q}$ decreases with increasing Froude number $Fr$ in all the pumping, free-pumping and co-pumping regions.

Figure 13: The Variation of Pressure Rise $\Delta p$ with $Q$ for Different Values of Froude Number $Fr$
with $\phi = 0.6$, $M = 1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $\lambda_1 = 0.3$ and $Da = 0.1$
The variation of pressure rise $\Delta p$ with $\bar{Q}$ for different values of Reynolds number $Re$ with $\phi = 0.6$, $M = 1$, $\alpha = \frac{\pi}{4}$, $\lambda_1 = 0.3$, $Fr = 2$ and $Da = 0.1$ is shown in Fig. 14. It is noticed that, the time averaged flux $\bar{Q}$ increases with an increase in Reynolds number $Re$ in all the pumping, free-pumping and co-pumping regions.

![Figure 14: The Variation of Pressure Rise $\Delta p$ with $\bar{Q}$ for Different Values of Reynolds Number $Re$ with $\phi = 0.6$, $M = 1$, $\alpha = \frac{\pi}{4}$, $\lambda_1 = 0.3$, $Fr = 2$ and $Da = 0.1$](image)

Figure 15 depicts the variation of pressure rise $\Delta p$ with $\bar{Q}$ for different values of amplitude ratio $\phi$ with $\lambda_1 = 0.3$, $M = 1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $Fr = 2$ and $Da = 0.1$. It is observed that, the time averaged flux $\bar{Q}$ increases with increasing $\phi$ in both pumping and free pumping regions, while it decreases with increasing $\phi$ in the co-pumping region for appropriately chosen $\Delta p$ ($= -19.35$).

![Figure 15: The Variation of Pressure Rise $\Delta p$ with $\bar{Q}$ for Different Values of Amplitude Ratio $\phi$ with $\lambda_1 = 0.3$, $M = 1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $Fr = 2$ and $Da = 0.1$](image)

The variation of frictional force $F$ with $\bar{Q}$ for different values of Darcy number $Da$ with $\phi = 0.6$, $M = 1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $Fr = 2$ and $\lambda_1 = 0.3$ is presented in Fig. 16. It is found that, the frictional force $F$ initially increases and then decreases with increasing Darcy number $Da$.

![Figure 16: The Variation of Frictional Force $F$ with $\bar{Q}$ for Different Values of Darcy Number $Da$ with $\phi = 0.6$, $M = 1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $Fr = 2$ and $\lambda_1 = 0.3$](image)
Figure 17 represents the variation of frictional force $F$ with $Q$ for different values of Hartmann number $M$ with $\phi = 0.6$, $Da = 0.1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $Fr = 2$ and $\lambda_1 = 0.3$. It is noted that, the frictional force $F$ initially decreases and then decreases with an increase in Hartmann number $M$.

![Figure 17](image1.png)

Figure 17: The Variation of Frictional Force $F$ with $Q$ for Different Values of Hartmann Number $M$
with $\phi = 0.6$, $Da = 0.1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $Fr = 2$ and $\lambda_1 = 0.3$

The variation of frictional force $F$ with $Q$ for different values of $\lambda_1$ with $\phi = 0.6$, $M = 1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $Fr = 2$ and $Da = 0.1$ is depicted in Fig. 18. It is observed that, the frictional force $F$ first increases and then decreases with increasing $\lambda_1$.

![Figure 18](image2.png)

Figure 18: The Variation of Frictional Force $F$ with $Q$ for Different Values of $\lambda_1$
with $\phi = 0.6$, $M = 1$, $\alpha = \frac{\pi}{4}$, $Re = 10$, $Fr = 2$ and $Da = 0.1$

Figure 19 illustrates the variation of frictional force $F$ with $Q$ for different values of inclination angle $\alpha$ with $\phi = 0.6$, $M = 1$, $\lambda_1 = 0.3$, $Re = 10$, $Fr = 2$ and $Da = 0.1$. It is found that, the frictional force $F$ decreases with increasing inclination angle $\alpha$.

![Figure 19](image3.png)

Figure 19: The Variation of Frictional Force $F$ with $Q$ for Different Values of Inclination Angle $\alpha$
with $\phi = 0.6$, $M = 1$, $\lambda_1 = 0.3$, $Re = 10$, $Fr = 2$ and $Da = 0.1$
The variation of frictional force $F$ with $\bar{Q}$ for different values of Froude number $Fr$ with $\phi = 0.6$, $M = 1$, $\alpha = \frac{\pi}{4}$, Re = 10, $\lambda_1 = 0.3$ and $Da = 0.1$ is shown in Fig. 20. It is noticed that, the frictional force $F$ increases with an increase in Froude number $Fr$.

![Figure 20: The Variation of Frictional force $F$ with $\bar{Q}$ for Different Values of Froude Number $Fr$
with $\phi = 0.6$, $M = 1$, $\alpha = \frac{\pi}{4}$, Re = 10, $\lambda_1 = 0.3$ and $Da = 0.1$](image1)

Figure 21 shows the variation of frictional force $F$ with $\bar{Q}$ for different values of Reynolds number Re with $\phi = 0.6$, $M = 1$, $\alpha = \frac{\pi}{4}$, $\lambda_1 = 0.3$, $Fr = 2$ and $Da = 0.1$. It is observed that, the frictional force $F$ decreases with increasing Reynolds number Re.

![Figure 21: The Variation of Frictional Force $F$ with $\bar{Q}$ for Different Values of Reynolds Number Re
with $\phi = 0.6$, $M = 1$, $\alpha = \frac{\pi}{4}$, $\lambda_1 = 0.3$, $Fr = 2$ and $Da = 0.1$](image2)

The variation of frictional force $F$ with $\bar{Q}$ for different values of amplitude ratio $\phi$ with $\lambda_1 = 0.3$, $M = 1$, $\alpha = \frac{\pi}{4}$, Re = 10, $Fr = 2$ and $Da = 0.1$ is depicted in Fig. 22. It is found that, the frictional force first decreases and then increases with an increase in amplitude ratio $\phi$.

![Figure 22: The Variation of Frictional Force $F$ with $\bar{Q}$ for Different Values of Amplitude Ratio
$\phi$ with $\lambda_1 = 0.3$, $M = 1$, $\alpha = \frac{\pi}{4}$, Re = 10, $Fr = 2$ and $Da = 0.1$](image3)
5. CONCLUSIONS

In this paper, we studied the effect of magnetic field on peristaltic flow of a Jeffrey fluid through a porous medium in an inclined tube under the assumption of long wavelength. The expressions for the velocity field are pressure gradient are obtained analytically. The pressure gradient and the time-averaged flow rate are increases with increasing Hartmann number $M$, inclination angle $\alpha$, Reynolds number $Re$ and amplitude ratio $\phi$, while they decreases with increasing Darcy number $Da$, material parameter $\lambda_1$ and Froude number $Fr$. The frictional force first increases and then decreases with increasing Darcy number $Da$. The frictional force first decreases and then increases with increasing Hartmann number $M$ and amplitude ratio $\phi$. The frictional force decreases with increasing inclination angle $\alpha$ and Reynolds number $Re$, while it increases with increasing Froude number $Fr$.

REFERENCES