Determination of Liquefaction Susceptibility of Soil: A Least Square Support Vector Machine Approach

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ABSTRACT: This study employs Least Square Support Vector Machine (LSSVM) for determination of liquefaction susceptibility of soil. LSSVM model uses dataset from the 1999 Chi-Chi, Taiwan earthquake. This article uses LSSVM as a classification tool. Using cone resistance ($q_c$) and cyclic stress ratio (CSR), model has been developed for prediction of liquefaction susceptibility using LSSVM. Further an attempt has been made to simplify the model, requiring only two parameters [$q_c$ and maximum horizontal acceleration ($a_{max}$)], for prediction of liquefaction. Further, developed LSSVM model has been applied to different case histories available globally and the results obtained confirm the capability of LSSVM model. For Chi-Chi earthquake, the model predicts with accuracy of 100%, and in the case of global data, LSSVM model predicts with accuracy of 88%. The developed LSSVM model gives equations for determination of liquefaction susceptibility of soil. This study shows that LSSVM is a robust tool for determination of liquefaction susceptibility of soil.

Keywords: Earthquake; Cone Penetration Test; Liquefaction; Least Square Support Vector Machine.

1. INTRODUCTION

Liquefaction of soil is responsible for large amounts of damage in historical earthquakes around the world. Liquefaction is a phenomenon whereby a granular material (soil) transforms from a solid state to a liquefied state as a consequence of increase in pore water pressure. The effective stress of the soil therefore reduces causing loss of bearing capacity. Liquefaction of soils during the past earthquakes has resulted in building settlement and/or severe tilting, sand blows, lateral spreading, ground cracks, landslides, dam and high embankment failures and many other hazards. Therefore, the determination of liquefaction susceptibility of soil is an important task in geotechnical earthquake engineering. There are different methods available for determination of liquefaction susceptibility of soil based on standard penetration test (SPT) (Seed and Idriss, 1967; Seed and Idriss, 1971; Seed et al., 1983; Seed et al., 1984; Youd et al., 2001). Although the above SPT-based method
remains an important tool for evaluating liquefaction resistance, it has some drawbacks, primarily due to the variable nature of the SPT (Robertson and Campanella, 1985; Skempton, 1986).

The first cone penetration test (CPT)-based method for liquefaction evaluation was developed by Robertson and Campanella (1985). This method has been revised and updated by many researchers (Seed and de-Alba, 1986; Stark and Olson, 1995; Olsen, 1997; Robertson and Wride, 1998; Moss et al., 2006). Goh (1996) successfully used Artificial Neural Network (ANN) for determination of liquefaction susceptibility of soil based on CPT data. However, the ANN has some limitations such as black box approach, arriving at local minima, slow convergence speed, less generalization capability, etc (Park and Rilett, 1999; Kecman, 2001).

This study adopts Least Square Support Vector Machine (LSSVM) for prediction of liquefaction susceptibility of soil based on CPT data. LSSVM uses two models (MODEL I and MODEL II) for determination of liquefaction susceptibility of soil based on CPT data. This study uses the database collected by Ku et al. (2004) from Chi-Chi Earthquake, Taiwan. This data consists of a total of 46 liquefied sites and remaining 88 non-liquefied sites after the earthquake. LSSVM (Suykens and Vandewalle, 1999; Suykens et al., 2002) is a modification of the standard Support Vector Machine (SVM). The paper has the following aims:

• To examine the capability of LSSVM for prediction of liquefaction susceptibility of soil based on CPT data.
• To develop equation for determination of liquefaction susceptibility of soil based on the LSSVM.
• To determine the generalization capability of the developed LSSVM for global data (Goh, 1996).

2. DETAILS OF LSSVM

The LSSVM is a statistical learning method which has a self-contained basis of statistical-learning theory and excellent learning performance (Suykens et al., 2002). A binary classification problem is considered having a set of training vectors (D) belonging to two separate classes.

\[
D = \{ (x^1 , y^1 ) , ......., (x^n , y^n ) \} \quad x \in \mathbb{R}^n , \ y \in \{-1, +1\} \quad (1)
\]

Where \( x \in \mathbb{R}^n \) is an n-dimensional data vector with each sample belonging to either of two classes labelled as \( y \in \{-1, +1\} \), and \( n \) is the number of training data. For MODEL I, the input parameters are Cyclic Stress Ratio (CSR) and cone resistance (\( q_c \)). So, for MODEL I, \( x = [\text{CSR}, q_c] \). MODEL II uses maximum horizontal acceleration (\( a_{\text{max}} \)) and \( q_c \) as input parameters. Therefore, For MODEL II, \( x = [a_{\text{max}}, q_c] \). In the current context of classifying soil condition during earthquake, the two classes
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labeled as (+1, -1) may mean non liquefaction and liquefaction. The SVM approach aims at constructing a classifier of the form:

\[ y(x) = \text{sign}\left[ \sum_{k=1}^{N} \alpha_k y_k \psi(x, x_k) + b \right] \]  

(2)

Where, \( \alpha_k \) are positive real constants, \( b \) is a real constant and \( \psi(x, x_k) \) is kernel function and sign is the signum function. It gives +1 if the element is greater than or equal to zero and -1 if it is less than zero.

For the case of two classes, one assumes

\[ w^T \phi(x_k) + b \geq 1, \text{ if } y_k = +1 \text{ (No Liquefaction)} \]
\[ w^T \phi(x_k) + b \leq 1, \text{ if } y_k = -1 \text{ (Liquefaction)} \]  

(3)

Which is equivalent to

\[ y_k [w^T \phi(x_k) + b] \geq 1, \text{ } k = 1, \ldots, N \]  

(4)

Where \( \phi(\cdot) \) is a nonlinear function which maps the input space into a higher dimensional space. According to the structural risk minimization principle, the risk bound is minimized by formulating the following optimization problem:

Minimize: \[ \frac{1}{2} w^T w + \frac{\gamma}{2} \sum_{k=1}^{N} e_k^2 \]

Subjected to: \[ y_k [w^T \phi(x_k) + b] = 1 - e_k, \text{ } k = 1, \ldots, N \]  

(5)

Where, \( \gamma \) is the regularization parameter, determining the trade-off between the fitting error minimization and smoothness and \( e_k \) is error variable.

In order to solve the above optimization problem (Equation 5), the Lagrangian is constructed as follows:

\[ L(w, b, e, \alpha) = \frac{1}{2} w^T w + \frac{\gamma}{2} \sum_{k=1}^{N} e_k^2 - \sum_{k=1}^{N} \alpha_k \left[ y_k [w^T \phi(x_k) + b] - 1 + e_k \right] \]  

(6)

Where \( \alpha_k \) are Lagrange multipliers, which can be either positive or negative due to the equality constraints as follows from the Kuhn-Tucker conditions (Fletcher, 1987). The solution to the constrained optimization problem is determined by the saddle point of the Lagrangian function \( L(w, b, e, \alpha) \), which has to be minimized with respect to \( w, b, e_k \), and \( \alpha_k \). Thus, differentiating \( L(w, b, e, \alpha) \) with respect to \( w, b, e_k \) and \( \alpha_k \) and setting the results equal to zero, the following three conditions have been obtained:
\[ \frac{\partial L}{\partial w} = 0 \Rightarrow w = \sum_{k=1}^{N} \alpha_k y_k \phi(x_k) \]

\[ \frac{\partial L}{\partial b} = 0 \Rightarrow \sum_{k=1}^{N} \alpha_k y_k = 0 \]

\[ \frac{\partial L}{\partial e_k} = 0 \Rightarrow \alpha_k = y_k \]

\[ \frac{\partial L}{\partial \alpha_k} = 0 \Rightarrow y_k \left[ w^T \phi(x_k) + b \right] - 1 + e_k = 0, \quad k = 1, \ldots, N \quad (7) \]

The above equitation (7) can be written immediately as the solution to the following set of linear equations (Fletcher, 1987)

\[
\begin{bmatrix}
I & 0 & 0 & -Z^T \\
0 & 0 & 0 & -Y^T \\
0 & 0 & \gamma I & -I \\
Z & Y & I & 0
\end{bmatrix}
\begin{bmatrix}
w \\
b \\
e \\
\alpha
\end{bmatrix}
= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (8)
\]

Where \( Z = \begin{bmatrix} \phi(x_1)^T y_1; \cdots; \phi(x_N)^T y_N \end{bmatrix}, Y = [y_1; \cdots; y_N], I = [1; \cdots; 1], e = [e_1; \cdots; e_N], \alpha = [\alpha_1; \cdots; \alpha_N] \)

The solution is given by

\[
\begin{bmatrix}
0 & -Y^T \\
Y & \Omega + \gamma^{-1} I
\end{bmatrix}
\begin{bmatrix}
b \\
\alpha
\end{bmatrix}
= \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (9)
\]

Where \( \Omega = Z^T Z \) and the kernel trick can be applied within the \( \Omega \) matrix.

\( \Omega_{kl} = y_k y_l \phi(x_k)^T \phi(x_l) = y_k y_l K(x_k, x_l), \quad k, l = 1, \ldots, N. \quad (10) \)

Where \( K(x_k, x_l) \) is kernel function.

The classifier in the dual space takes the form

\[ y(x) = \text{sign} \left[ \sum_{k=1}^{N} \alpha_k y_k K(x, x_k) + b \right] \quad (11) \]

The above methodology has been used for prediction of liquefaction susceptibility of soil based on CPT data by developed two models (MODEL I and
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MODEL II). This study uses the database collected by Ku et al. (2004) from Chi-Chi Earthquake, Taiwan. This data consists of a total of 46 liquefied sites and remaining 88 non-liquefied sites after the earthquake. In the MODEL I, CSR versus q_c data is trained for prediction of liquefaction susceptibility. In the MODEL-II, this is further simplified by relating a_max versus q_c for prediction of liquefaction susceptibility. For the purpose of classification, a value of 1 is assigned to those sites that did not liquefy while a value of -1 is assigned to those sites that liquefied. The data has been further divided into two sub-sets; a training dataset, to construct the model, and a testing dataset to estimate the model performance. So, for our study a set of 94 data is considered as the training dataset and remaining set of 40 data is considered as the testing dataset. The data is normalized between 0 to 1. In this study, radial basis function is used as the kernel function of the LSSVM. When applying LSVM, the design value of width (σ) of radial basis function and γ will be determined during the modeling experiment. In MODEL I, CSR is used as one of the input parameters and it is a function of \( \sigma, \sigma', \) depth, magnitude of earthquake and \( a_{\text{max}} \) and it is defined as (Seed and Idriss, 1971):

\[
CSR = 0.65 \left( \frac{\sigma_v}{\sigma_v'} \right) \left( \frac{a_{\text{max}}}{g} \right) r_d
\]

(12)

Where, \( g \) is acceleration due to gravity and \( r_d \) is stress reduction. So, to calculate the value of CSR one has to determine the value of \( \sigma, \sigma', \) depth, magnitude of earthquake and \( a_{\text{max}} \). But, it is well known that CPT does not have provision to obtain soil sample and calculate the needed soil properties. For this reason, determination of CSR from CPT test alone is impossible. The purpose of the development of MODEL II is to simplify and predict the liquefaction based on q_c and \( a_{\text{max}} \). So, in MODEL II, the input variables are q_c and \( a_{\text{max}} \). In MODEL II, the same training dataset, testing dataset, normalization technique and same kernel function have been used as used in MODEL I. MODEL II has also been verified for additional 109 case histories (which were not part of training or testing dataset used earlier to develop the model) available globally (Goh, 1996).

3. RESULTS AND DISCUSSION

Training and testing performance has been determined by using the following equation.

\[
\text{Training or Testing performance} (\%) = \left( \frac{\text{No of data predicted accurately by LSSVM}}{\text{Total data}} \right) \times 100
\]

(13)

The design value of γ and σ has been determined by trial and error approach. For MODEL I, the design value of γ and σ is 50 and 5 respectively. The Performance
of training and testing dataset has been determined by the design value of $\gamma$ and $\sigma$. The performance of training and testing dataset is 100%. So, there is no misclassification for training and testing dataset. Fig. 1 shows the plot between $q_c$ and CSR for training and testing dataset. The following equation (by putting $K(x, x_k) = \exp\left\{-\frac{(x_k - x)(x_k - x)^T}{2\sigma^2}\right\}$ $\sigma = 5$, $b = 0.5301$ and $N = 94$ in equation 11) has been developed for the prediction of status of soil(s) during an earthquake.

$$s_{\text{MODEL I}} = \text{sign} \left[ \sum_{k=1}^{n} \alpha_k y_k \exp\left\{-\frac{(x_k - x)(x_k - x)^T}{2\sigma^2}\right\} + 0.5301 \right]$$

(14)

Fig. 2 shows the value of $a$ for MODEL.

User can use the equation (14) for prediction of liquefaction susceptibility of soil due to an earthquake.

For MODEL II, the design value of $\gamma$ and $\sigma$ is 50 and 5 respectively. The performance of training and testing is 100%. So, the performance of MODEL I and MODEL II are same.

![Figure 1: Plot between $q_c$ and CSR for MODEL I](image)
For MODEL II, the following equation (by putting
\[ K(x,x_k) = \exp \left\{ -\frac{(x_k - x)(x_k - x)^T}{2\sigma^2} \right\}, \sigma = 5, b = 1.118 \text{ and } N = 94 \text{ in equation 11} \) has been developed for prediction of susceptibility of soil due to an earthquake.

\[ s_{\text{MODEL II}} = \text{sign} \left[ \sum_{k=1}^{94} \alpha_k y_k \exp \left\{ -\frac{(x_k - x)(x_k - x)^T}{50} \right\} + 1.118 \right] \] (15)

The values of \( a \) in equation (15) have been given in Fig. 3.
The above developed equation (15) has been used in different case histories available globally (Goh, 1996). These global data are taken from five earthquakes which occurred in different countries in a period of about 20 years (1964–1983). The above equation has correctly classified 96 data out of 109 data. Therefore, the developed equation (15) can be used for prediction of liquefaction susceptibility of soil. Fig. 4 shows the plot between $q_c$ and $a_{\text{max}}$. Fig. 4 can be used as practical tool to classify between liquefiable and non-liquefiable soil.

Figure 4: Plot between $q_c$ and $a_{\text{max}}$

The developed MODEL II does not require the determination of Cyclic Stress Ratio (CSR) and Cyclic Resistance Ratio (CRR) for prediction of liquefaction susceptibility of soil.

4. CONCLUSION

This article successfully applied LSSVM for determination of liquefaction susceptibility of soil. It is clear from the result that, with an appropriate selection of $\gamma$ and $\sigma$ value, accuracy of the order of 100% has been achieved from LSSVM classification. The MODEL II presented clearly that only two parameters ($q_c$ and $a_{\text{max}}$) are sufficient input parameters for predicting liquefaction susceptibility of a site with depth. User can use the developed equations for determination of liquefaction susceptibility of soil. In summary, the developed LSSVM can be used as a practical tool for determination of liquefaction susceptibility of soil.
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References


